

CM4655 Polymer Rheology Lab

Notes on Capillary Flow Corrections

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(all page numbers reference *Understanding Rheology*, by F. A. Morrison, Oxford 2001)

There are three corrections that may be applied to capillary flow. They must be performed in the order: slip correction, entrance-pressure correction, non-parabolic velocity profile correction.

Part 1: Correction for Slip (Mooney Analysis, pp394-397)

When the liquid does not adhere to the walls, the velocity of the fluid at the wall makes the flow rate Q larger than it would be if there were no slip. The flow rate is related to the raw average velocity as

$$v_{raw} = \frac{Q}{\pi R^2}$$

The measured average velocity v_{raw} is related to the true average velocity v_{true} through the slip velocity.

$$v_{true} = v_{raw} - v_{slip}$$

If we multiply by $4/R$, we obtain the relationship between the true shear rate and the raw shear rate.

$$\begin{aligned} \dot{\gamma}_{raw} &= \frac{4Q}{\pi R^3} = \frac{4v_{raw}}{R} \\ \frac{4v_{true}}{R} &= \frac{4v_{raw}}{R} - \frac{4v_{slip}}{R} \end{aligned} \quad (1)$$

Rearranging, we obtain,

$$\begin{aligned} \frac{4Q}{\pi R^3} &= (slope) \frac{1}{R} + (intercept) \\ &= 4v_{slip} \frac{1}{R} + \frac{4v_{true}}{R} \end{aligned}$$

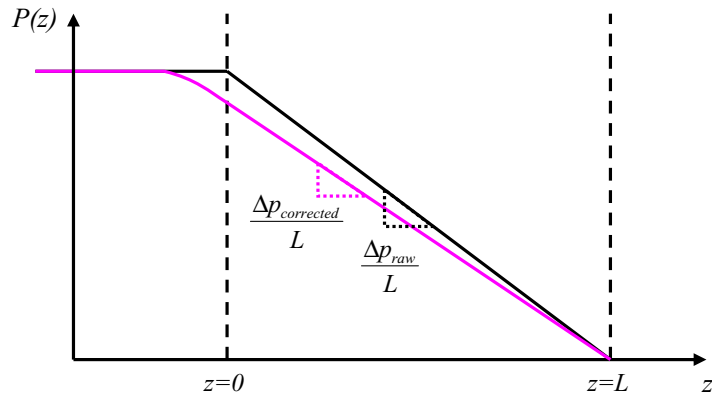
which when plotted as $\frac{4Q}{\pi R^3}$ versus $1/R$ is a line with slope equal to four times the slip velocity (see figure 10.10, p396).

Note that all data points should refer to the same wall shear stress $\Delta p R / 2L$. If the data are taken on the Goettfert, we specify the flow rate and measure the pressure. It will therefore be necessary most of the time to interpolate on the lines of Δp versus Q to arrive at points that are all at the same value of wall shear stress.

Once the slip velocities are calculated, the flow rates can be corrected through equation (1) above.

Part 2: Correction for Entrance and Exit Effects (Bagley Plot, pp393-394)

A correction to the measured pressure is often necessary due to losses that occur during the reduction in radius between the barrel and the capillary. The measured pressure is the pressure in the barrel. The raw pressure-drop is calculated as that pressure divided by the capillary length L . There is a loss of pressure at the entrance to the capillary, however, and thus the true pressure drop across the capillary is smaller than the raw pressure-drop. The entrance pressure losses must be subtracted from the raw pressure in order to calculate the correct pressure drop across the capillary.



The wall shear stress is given by

$$\tau_R = \frac{\Delta p R}{2L}$$

This can be rearranged as

$$\Delta p = 2\tau_R \frac{L}{R}$$

Thus, if data are taken on a variety of capillaries, a plot of Δp versus L/R at constant wall shear stress (which is the same as constant apparent shear rate at steady state) should go through the origin and have slope equal to $2\tau_R$ (see Figure 10.8, p394).

If the line does not go through the origin, this is a reflection of combined entrance and exit pressure losses. We can correct for these losses by subtracting the y-intercept Δp_{ent} of such a plot from the y-values.

$$\begin{aligned} \Delta p_{raw} &= (slope) \frac{L}{R} + (intercept) \\ &= 2\tau_R \frac{L}{R} + \Delta p_{ent} \\ \Delta p_{corrected} &= (\Delta p_{raw} - \Delta p_{ent}) = 2\tau_R \frac{L}{R} \\ \tau_R &= \frac{\Delta p_{corrected} R}{2L} \end{aligned}$$

The entrance/exit loss correction is thus a correction to the pressure measurement.

Part 3: Weissenberg-Rabinowitsch Correction for Nonparabolic Velocity Profile (pp 387-392)

The velocity profile of pressure-driven flow of a Newtonian fluid in a pipe is a parabola (equation 3.216).

$$v_z(r) = \frac{\Delta p R^2}{4\mu L} \left(1 - \frac{r^2}{R^2}\right)$$

Integration of this velocity profile results in the Hagen-Poiseuille law, the pressure-drop/flow-rate relationship for Newtonian fluids (equation 3.219).

$$Q = \frac{\Delta p \pi R^4}{8\mu L}$$

Thus, for Newtonian fluids, measurements of pressure-drop and flow rate in a capillary tube may be plotted as $4Q/\pi R^3$ (shear rate at the wall for a Newtonian fluid) versus $\Delta p R/2L$ (shear stress at the wall for all fluids), which is a straight line of slope equal to the inverse of the viscosity.

$$\frac{4Q}{\pi R^3} = \frac{1}{\mu} \frac{\Delta p R}{2L}$$

The quantity $4Q/\pi R^3$ is therefore called the apparent shear rate $\dot{\gamma}_a$, meaning it is the shear rate one could deduce from the flow rate, presuming that the fluid is Newtonian.

For non-Newtonian fluids, the shear rate will be different than $\dot{\gamma}_a$ (see Figure 10.6). The correct shear rate at the wall for a non-Newtonian fluid may be calculated from equation 10.44.

$$\dot{\gamma}_R = \dot{\gamma}_a \left[\frac{1}{4} + 3 \frac{d \ln \dot{\gamma}_a}{d \ln \tau_R} \right]$$

The quantity in square brackets is the Weissenberg-Rabinowitsch (WR) correction, which is derived in the text.

To calculate the Weissenberg-Rabinowitsch calculation from data, a plot of $\ln \dot{\gamma}_a$ versus $\ln \tau_R$ is made. If the line is straight, a regression yields the needed term in the WR correction. If the line is not straight, it can usually be fit with a polynomial.

$$y = (\ln \dot{\gamma}_a)$$

$$x = (\ln \tau_R)$$

$$y = ax^2 + bx + c$$

From the polynomial we can calculate the needed derivative.

$$\frac{dy}{dx} = 2ax + b$$

For every data point $(\tau_R, \dot{\gamma}_R)$ the WR correction can then be calculated and applied.

Once the apparent shear rate is corrected to the true shear rate, the viscosity may be calculated as the ratio of the shear stress at the wall and the true shear rate at the wall of the capillary.

$$\eta = \frac{\tau_R}{\dot{\gamma}_R}$$