CM4655 Polymer Rheology Lab

Elongational Flow Measurement

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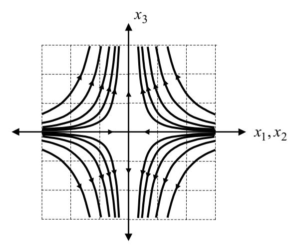

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Two main standard flows in rheology:

Shear Flow capillary flow; torsional flow

Elongational Flow die entry flow

this is next

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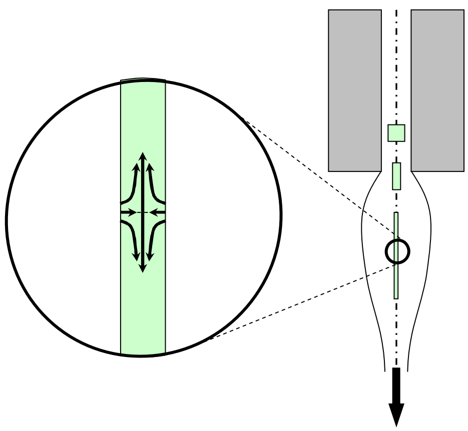
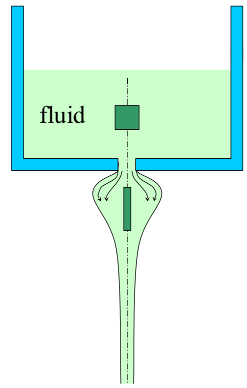
Elongational flow occurs when there is stretching - die exit, flow through contractions





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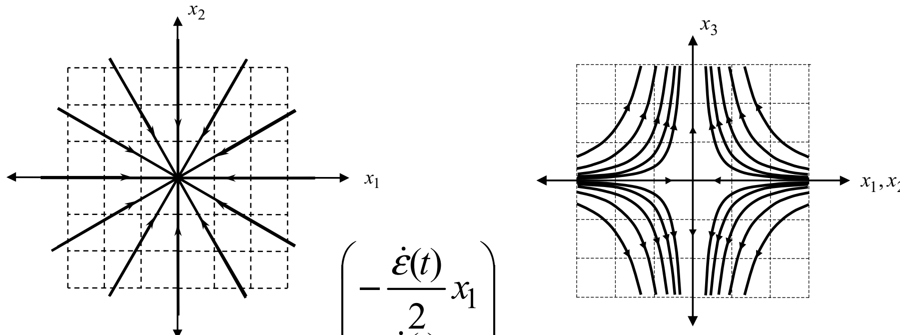
Uniaxial Elongational Flow



$$\underline{v} \equiv \begin{pmatrix} -\frac{\dot{\varepsilon}(t)}{2} x_1 \\ -\frac{\dot{\varepsilon}(t)}{2} x_2 \\ \dot{\varepsilon}(t) x_3 \end{pmatrix}_{123} \qquad \dot{\varepsilon}(t) > 0$$

path lines

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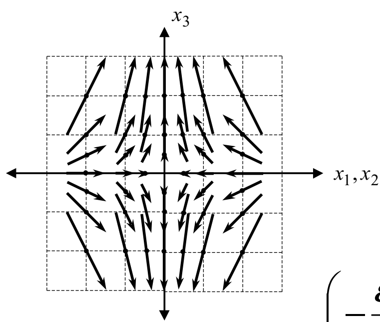
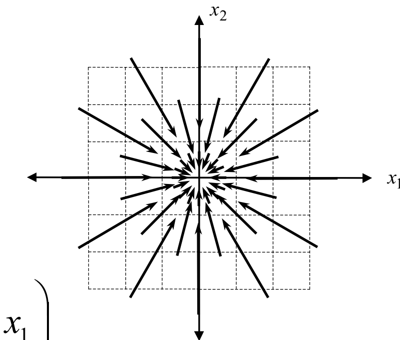
Uniaxial Elongational Flow





$$\underline{v} \equiv \begin{pmatrix} -\frac{\dot{\varepsilon}(t)}{2} x_1 \\ -\frac{\dot{\varepsilon}(t)}{2} x_2 \\ \dot{\varepsilon}(t) x_3 \end{pmatrix}_{123} \qquad \dot{\varepsilon}(t) > 0$$

velocity field

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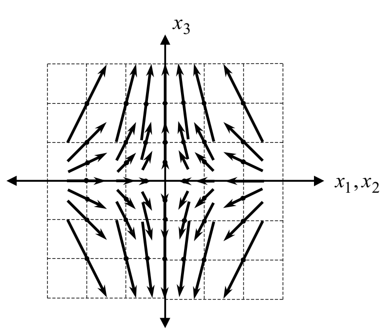
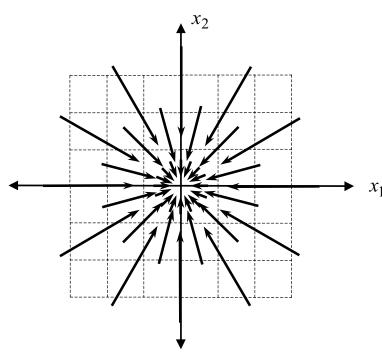
How does the stress tensor simplify for elongational flow?





There is 180° of symmetry around all three coordinate axes.

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Because of symmetry, there are only 3 nonzero components of the extra stress tensor in elongational flows.

ELONGATION:

$$\underline{\underline{\tau}} = \begin{pmatrix} \tau_{11} & 0 & 0 \\ 0 & \tau_{22} & 0 \\ 0 & 0 & \tau_{33} \end{pmatrix}_{123}$$

This greatly simplifies the experimentalists tasks as only three stress components must be measured instead of 6.

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Steady Elongational Flow Material Functions

Kinematics:

$$\underline{v} = \begin{pmatrix} -\frac{1}{2}\dot{\varepsilon}(t)(1+b)x_1 \\ -\frac{1}{2}\dot{\varepsilon}(t)(1-b)x_2 \\ \dot{\varepsilon}(t)x_3 \end{pmatrix}_{123}$$

$$\dot{\varepsilon}(t) = \dot{\varepsilon}_0 = \text{constant}$$

Elongational flow: b=0, $\dot{\varepsilon}(t) > 0$

Biaxial stretching: b=0, $\dot{\varepsilon}(t) < 0$

Planar elongation: b=1, $\dot{\varepsilon}(t) > 0$

Material Functions:

$$\bar{\eta} \text{ or } \bar{\eta}_B \text{ or } \bar{\eta}_{P_1} \equiv \frac{-(\tau_{33} - \tau_{11})}{\dot{\varepsilon}_0}$$

Uniaxial or Biaxial or First Planar Elongational Viscosity

$$\bar{\eta}_{P_2} \equiv \frac{-(\tau_{22} - \tau_{11})}{\dot{\varepsilon}_0}$$

Second Planar Elongational Viscosity

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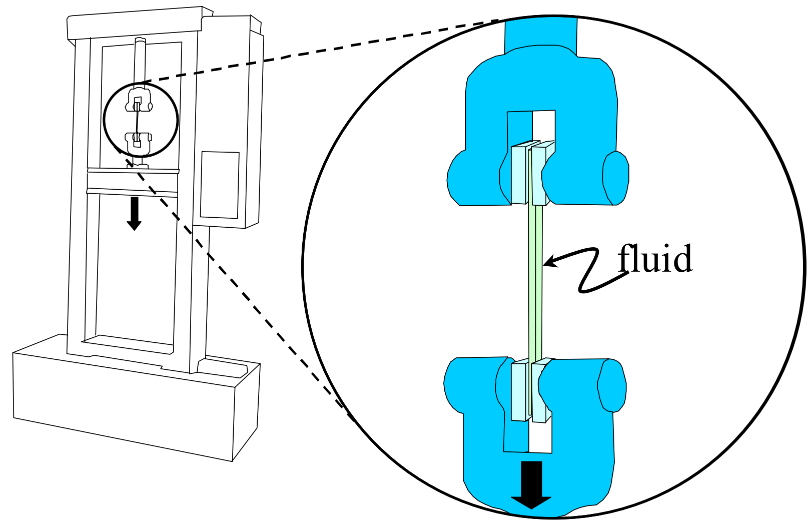

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Uniaxial Extension

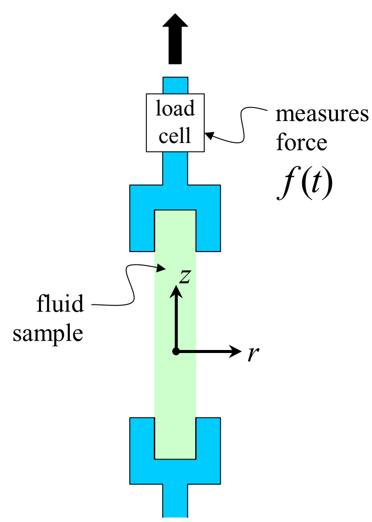
tensile force

$$\tau_{zz} - \tau_{rr} = -\frac{f(t)}{A(t)}$$

time-dependent cross-sectional area

For homogeneous flow: $A(t) = A_0 e^{-\varepsilon_0 t}$

$$\bar{\eta} = \frac{-(\tau_{zz} - \tau_{rr})}{\varepsilon_0} = \frac{f(t_\infty) e^{\varepsilon_0 t_\infty}}{A_0 \varepsilon_0}$$



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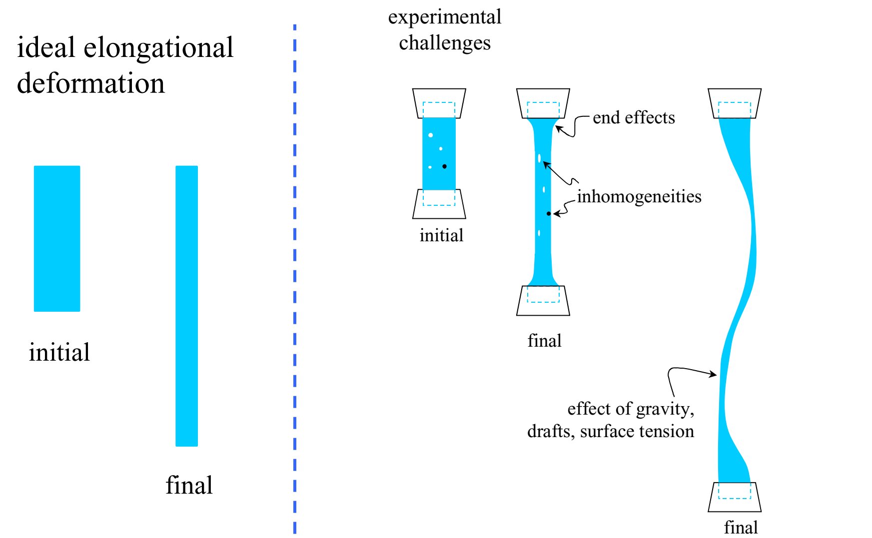
Experimental Difficulties in Elongational Flow



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Several specialized elongational rheometers have been developed and commercialized over the last 20 years

1. *Filament Stretching Elongational Rheometer (FiSER)*
2. *Metal Belt Elongational Rheometer (MBER)*
3. *Sentmanat Extension Rheometer (SER)*
4. *Capillary Breakup Elongational Rheometer (CaBER)*

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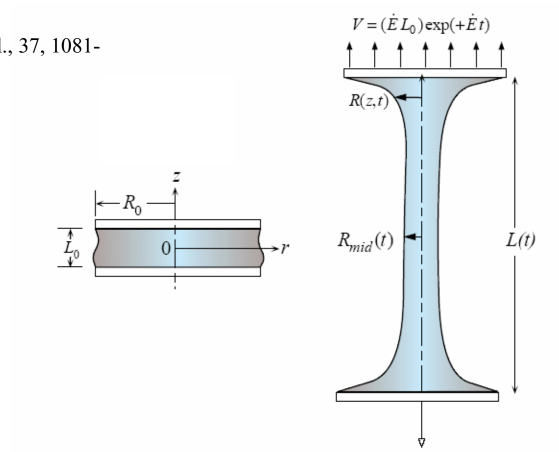
Filament Stretching Rheometer (FiSER)

Tirtaatmadja and Sridhar, J. Rheol., 37, 1081-1102 (1993)

- Optically monitor the midpoint size
- Very susceptible to environment
- End Effects



McKinley, et al., 15th Annual Meeting of the International Polymer Processing Society, June 1999.

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Rheol Acta (2001) 40: 457–466
© Springer-Verlag 2001

ORIGINAL CONTRIBUTION

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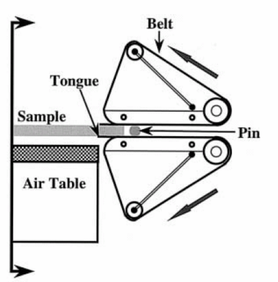
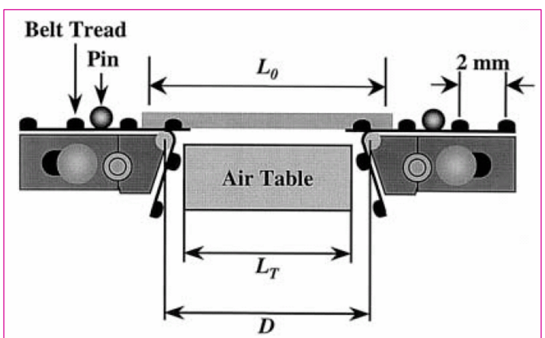
A comparison of extensional viscosity measurements from various RME rheometers

- Steady and startup flow
- Recovery
- Good for melts





RHEOMETRICS RME

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Conclusions

Extensional viscosity measurements of a slightly strain hardening LLDPE (Dow Affinity PL 1880) from several Rheometric Scientific RME extensional rheometers were compared with data obtained from the original version of the RME at the ETH Institut für Polymere in Zürich and the Münstedt Tensile Rheometer (MTR) at the University of Erlangen. In general, the commercial RMEs extended samples with a strain rate that was significantly less than the set strain rate. The problem worsened at the higher strain rates of 1.0 s^{-1} and 0.1 s^{-1}, where the difference was at least 10%. The data from the commercial RMEs typically agree with the MTR and original RME within 20%, after the extensional viscosity is corrected for the strain rate.

Achieving commanded strain requires great care.

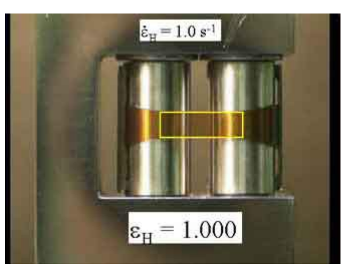
increased from 50 mm to 60 mm, the deviation in the strain rate decreased from 20% to 2–6%. The recommended value of L_0 should be determined by measuring the distance D and using Eq. (4). However, operating the RME with the correct value of L_0 does not eliminate entirely the strain rate deviation. Based on the performance of earlier rotary clamp rheometers, the strain rate deviation most likely occurs because the velocity of the belts is not sufficiently transferred to the sample during the test. Clearly, the deformation of all materials must be monitored with a video camera, and analyzed to obtain the true strain rate applied to the sample during the test.

Extensional viscosity data in the first 1–2 s of the test improved at all strain rates with the use of pins and by

Use of the video camera (although tedious) is recommended in order to get correct strain rate.

Sentmanat Extension Rheometer

- Originally developed for rubbers, good for melts
- Measures elongational viscosity, startup, other material functions
- Two counter-rotating drums
- Easy to load; reproducible



www.xpansioninstruments.com

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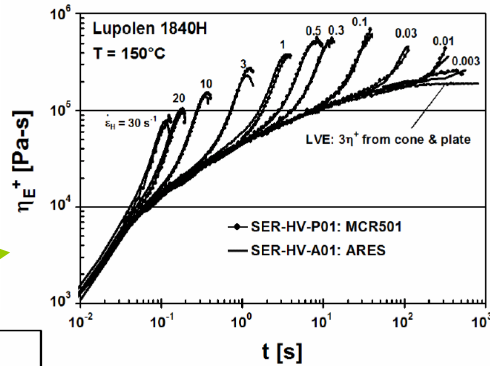
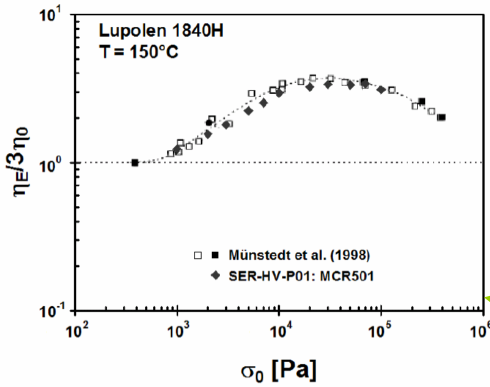
Sentmanat et al., J. Rheol., 49(3) 585 (2005)

Comparison on different host instruments





Comparison with other instruments (literature)

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CaBER Extensional Rheometer

- •Polymer solutions
- •Works on the principle of capillary filament break up
- •Cambridge Polymer Group and HAAKE

For more on theory see: campoly.com/notes/007.pdf

Brochure: www.thermo.com/com/cda/product/detail/1,,17848,00.html

Operation

- •Impose a rapid step elongation
- •form a fluid filament, which continues to deform
- •flow driven by surface tension
- •also affected by viscosity, elasticity, and mass transfer
- •measure midpoint diameter as a function of time
- •Use force balance on filament to back out an apparent elongational viscosity

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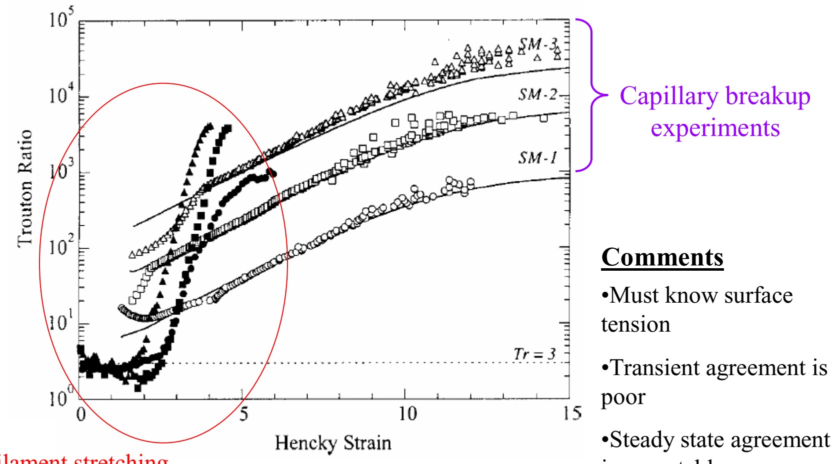

Comments

- Must know surface tension
- Transient agreement is poor
- Steady state agreement is acceptable
- Be aware of effect modeling assumptions on reported results

Anna and McKinley, *J. Rheol.* 45, 115 (2001).

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We do not have an elongational rheometer

We can estimate an elongational viscosity with capillary results

Die Entry Flow

Cogswell Analysis

Binding Analysis

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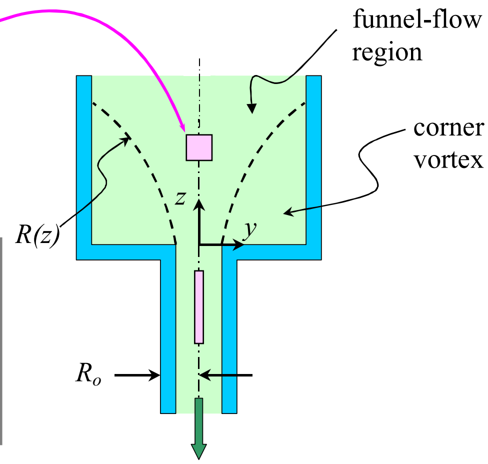
Elongational Viscosity via Contraction Flow: Cogswell/Binding Analysis



By making strong assumptions about the flow we can relate the pressure drop across the contraction to an elongational viscosity

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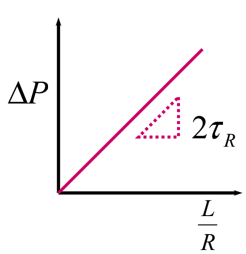
Entrance and exit effects - Bagley correction

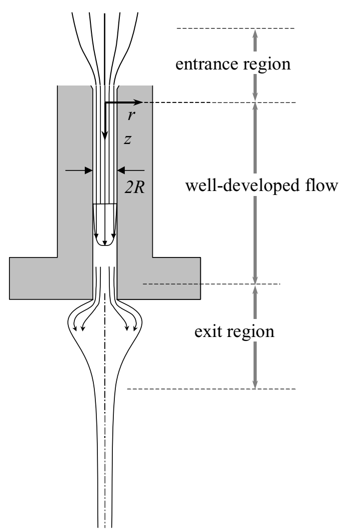
$$\tau_R = \frac{\Delta P R}{2L} \Rightarrow \Delta P = (2\tau_R)\frac{L}{R}$$

Constant at constant Q

Run for different capillaries



This is the result when the end effects are negligible.



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Bagley Plot

$$\Delta P_{\substack{end\\effects}} = f(Q) = f(\dot{\gamma}_a)$$

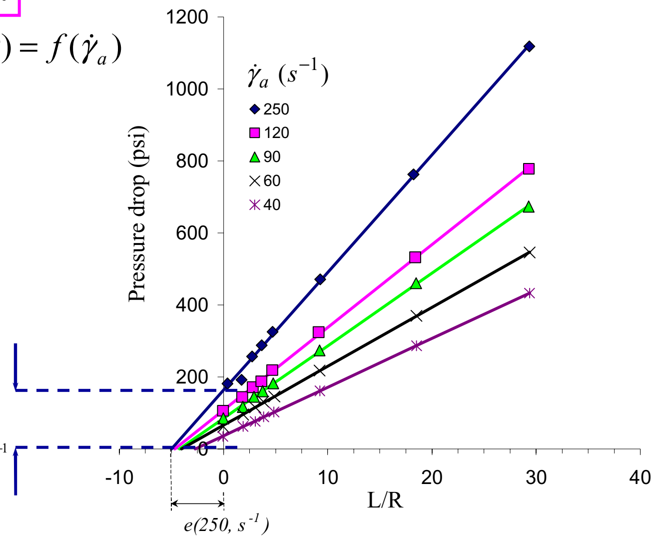

Figure 10.8, p. 394 Bagley, PE

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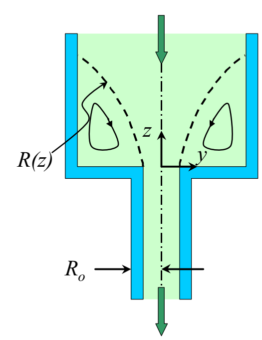
Assumptions for the Cogswell Analysis

- incompressible fluid
- funnel-shaped flow; no-slip on funnel surface
- unidirectional flow in the funnel region
- well developed flow upstream and downstream
- θ-symmetry
- pressure drops due to shear and elongation may be calculated separately and summed to give the total entrance pressure-loss
- neglect Weissenberg-Rabinowitsch correction
- shear stress is related to shear-rate through a power-law
- elongational viscosity is constant
- shape of the funnel is determined by the minimum generated pressure drop
- no effect of elasticity (shear normal stresses neglected)
- neglect inertia



$$\dot{\gamma} \approx \dot{\gamma}_a$$
$$\underline{\tau}_R = m\dot{\gamma}_a^n$$
$$\bar{\eta} = \text{constant}$$

F. N. Cogswell, Polym. Eng. Sci. (1972) 12, 64-73.
F. N. Cogswell, Trans. Soc. Rheol. (1972) 16, 383-403.

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Cogswell Analysis

elongation rate $\dot{\varepsilon}_o = \frac{\tau_R \dot{\gamma}_a}{2(\tau_{11}-\tau_{22})}$

$\tau_R = \eta\,\dot{\gamma}_a$ $\qquad \eta = m\dot{\gamma}_a^{n-1}$

$\dot{\gamma}_a = \frac{4Q}{\pi R^3}$

elongation normal stress $(\tau_{11}-\tau_{22}) = -\frac{3}{8}\Delta p_{ent}\,(n+1)$

elongation viscosity $\bar{\eta} \approx \frac{-(\tau_{11}-\tau_{22})}{\dot{\varepsilon}_o} = \frac{\frac{9}{32}(n+1)^2 \Delta p_{ent}^2}{\tau_R \dot{\gamma}_a}$

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Cogswell Analysis – using Excel

From shear: $\eta = m\dot{\gamma}_a^{n-1}$

$\dot{\varepsilon}_o = \frac{\tau_R \dot{\gamma}_a}{2(\tau_{11}-\tau_{22})}$ $\qquad \bar{\eta} = \frac{-(\tau_{11}-\tau_{22})}{\dot{\varepsilon}_o}$ $\qquad 3\eta$

$\dot{\gamma}_a = \frac{4Q}{\pi R^3}$	Δp_{ent}	Δp_{ent}	τ_R				
RAW DATA gammdotA	RAW DATA deltPent(psi)	deltPent(Pa)	sh stress(Pa)	N1(Pa)	e_rate	Cogswell elongvisc	Cogswell 3*shearVisc
250	163.53	1.13E+06	1.13E+05	-6.27E+05	2.25E+01	2.79E+04	1.55E+03
120	107.72	7.43E+05	7.92E+04	-4.13E+05	1.15E+01	3.59E+04	2.27E+03
90	85.311	5.88E+05	6.95E+04	-3.27E+05	9.56E+00	3.42E+04	2.65E+03
60	66.018	4.55E+05	5.64E+04	-2.53E+05	6.69E+00	3.79E+04	3.23E+03
40	36.81	2.54E+05	4.65E+04	-1.41E+05	6.59E+00	2.14E+04	4.00E+03

$(\tau_{11}-\tau_{22}) = -\frac{3}{8}\Delta p_{ent}(n+1)$

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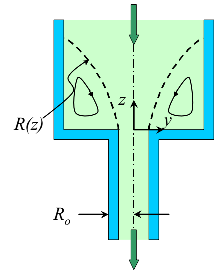
Assumptions for the Binding Analysis

- incompressible fluid
- funnel-shaped flow; no-slip on funnel surface
- unidirectional flow in the funnel region
- well developed flow upstream and downstream
- θ -symmetry
- shear viscosity is related to shear-rate through a power-law
- elongational viscosity is given by a power law
- shape of the funnel is determined by the minimum work to drive flow
- no effect of elasticity (shear normal stresses neglected)
- the quantities $(dR/dz)^2$ and d^2R/dz^2, related to the shape of the funnel, are neglected; implies that the radial velocity is neglected when calculating the rate of deformation
- neglect energy required to maintain the corner circulation
- neglect inertia



$$\tau_R = m\dot{\gamma}_a^n$$
$$\bar{\eta} = l\dot{\varepsilon}_o^{\,t-1}$$

D. M. Binding, JNNFM (1988) 27, 173-189.

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Binding Analysis

l, **elongational prefactor**

$$\Delta p_{ent} = \frac{2m(1+t)^2}{3t^2(1+n)^2}\left\{\frac{lt(3n+1)n^t I_{nt}}{m}\right\}^{1/(1+t)} \dot{\gamma}_{R_o}^{\;t(n+1)/(1+t)}\left\{1-\alpha^{3t(n+1)/(1+t)}\right\}$$

$$I_{nt} = \int_0^1 \left|2-\left(\frac{3n+1}{n}\right)\phi^{1+1/n}\right|^{t+1}\phi\, d\phi$$

$$\eta = m\dot{\gamma}_a^{n-1}$$

$$\dot{\gamma}_{R_o} = \frac{(3n+1)\,Q}{n\pi R_o^3}$$

$$\alpha = \frac{R_0(capillary)}{R_1(barrel)}$$

elongation viscosity $\quad \bar{\eta} = l\dot{\varepsilon}_o^{\,t-1}$

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Binding Analysis

Note: there is a non-iterative solution method described in the text; The method using Solver is slightly preferable, since it uses all the data in finding optimal values of l and t.

Evaluation Procedure

1. Shear power-law parameter n must be known; must have data for Δp_{ent} versus Q
2. Guess t, l
3. Evaluate I_{nt} by numerical integration over ϕ
4. Using Solver, find the best values of t and l that are consistent with the Δp_{ent} versus Q data

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Binding Analysis – using Excel Solver

Evaluate integral numerically

$$I_{nt} = \int_0^1 \left| 2 - \left(\frac{3n+1}{n} \right) \phi^{1+1/n} \right|^{t+1} \phi \, d\phi$$

phi	f(phi)	areas
0	0	
0.005	0.023746502	5.93663E-05
0.01	0.047492829	0.000178098
0.015	0.071238512	0.000296828
0.02	0.094982739	0.000415553
0.025	0.118724352	0.000534268
0.03	0.142461832	0.000652965
0.035	0.166193303	0.000771638
0.04	0.189916517	0.000890275
0.045	0.213628861	0.001008863
0.05	0.237327345	0.001127391
0.055	0.261008606	0.00124584
0.06	0.2846689	0.001364194
0.065	0.308304107	0.001482433

. . .

$$area = \frac{1}{2}(b_1 + b_2)h$$

Summing:

Int= 1.36055

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Binding Analysis – using Excel Solver

Optimize t, l using Solver

By varying these cells:

$$\frac{(predicted - actual)^2}{(actual)^2}$$

t_guess=	1.2477157	
l_guess=	11991.60895	
******* SOLVER SOLUTION ********		
predicted DeltaPent	exptal DeltaPent	difference
1.26E+06	1.13E+06	1.35E-02
6.88E+05	7.43E+05	5.51E-03
5.43E+05	5.88E+05	6.02E-03
3.89E+05	4.55E+05	2.14E-02
2.78E+05	2.54E+05	9.28E-03
	target cell	5.57E-02

Sum of the differences: Minimize this cell

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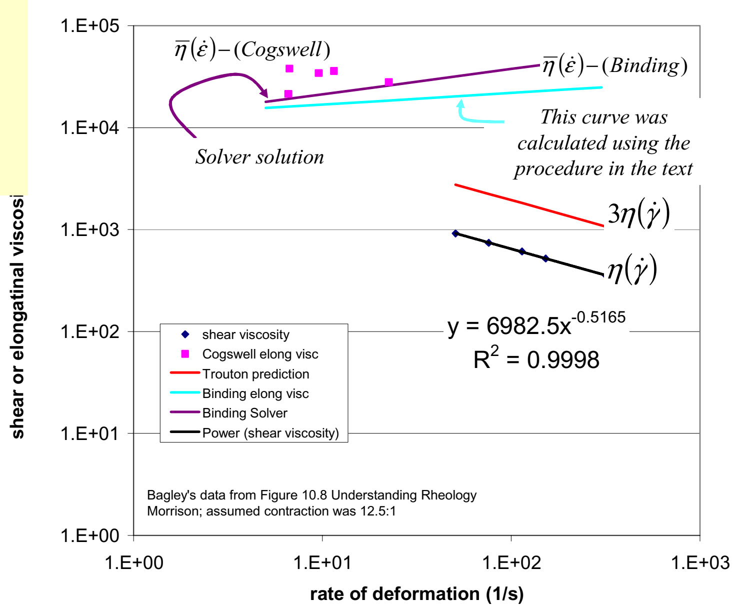
Example calculation from Bagley's Data



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Assignment:

Estimate the elongational viscosity of your polymer as a function of temperature. Compare your results with Trouton's rule.

Trouton's Rule	$\bar{\eta} = 3\eta$

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