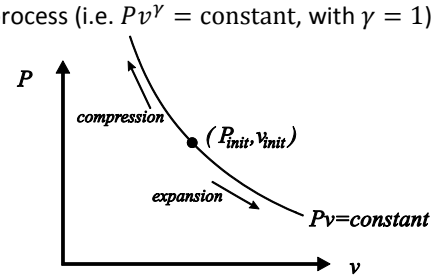
CM 3230 Thermodynamics, Fall 2016

Lecture 4

1. Reversible processes for <u>ideal gases</u> in closed system (section 2.7)

a) **<u>Isothermal Expansion and Compression</u>** ($\Delta u = 0$ because $u = u(T)$ for ideal gas)

- Path in P vs v diagram
  i. From Ideal gas law, for isothermal process, $Pv = RT = \text{constant}$
  ii. It is a polytropic process (i.e. $Pv^{\gamma} = \text{constant}$, with $\gamma = 1$)



- Reversible work of isothermal process

$$w_{by,1\to2} = \int P\,dv = RT\int \frac{1}{v}\,dv = RT\ln\frac{v_2}{v_1}$$

$$\text{or } w_{by,1\to2} = RT\ln\frac{P_1}{P_2}$$

- Heat in: $q_{in} = w_{by,1\to2} = RT\ln(P_1/P_2)$

b) <u>Adiabatic Expansion and Compression</u>

For ideal gases, $dv = d\left(\frac{RT}{P}\right) = R\left(\frac{1}{P}dT - \frac{T}{P^2}dP\right)$

- Path in P vs v diagram
  i. Adiabatic → $q_{in} = 0$, thus

$$(du = c_v dT) = -\delta w_{by} = -Pdv$$

$$c_v dT = -R\left(dT - \frac{T}{P}dP\right)$$

Rearranging,

$$\frac{c_v + R}{R}\frac{dT}{T} = \frac{dP}{P}$$

Assume c_v constant (not true for all ideal gases), then integrating,

$$\left(\frac{c_v + R}{R}\right)\ln\frac{T_2}{T_1} = \ln\frac{P_2}{P_1} \quad \rightarrow \quad \left(\frac{T_2}{T_1}\right)^{(c_v+R)/R} = \left(\frac{P_2}{P_1}\right)$$

Since $c_P = c_v + R$, we have $(c_v + R)/R = c_p/(c_p - c_v) = k/(k-1)$, where $k = c_p/c_v$. Thus,

$$\left(\frac{T_2}{T_1}\right)^{k/(k-1)} = \left(\frac{P_2}{P_1}\right)$$

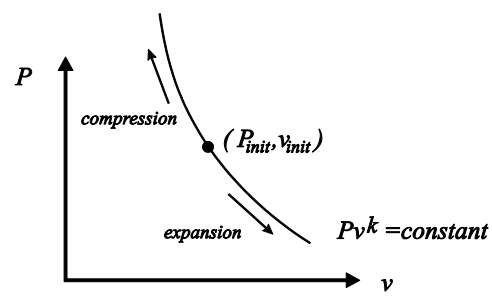
To obtain P vs v relationship, use ideal gas law to replace T,

$$\left(\frac{P_2 v_2}{P_1 v_1}\right)^{k/(k-1)} = \left(\frac{P_2}{P_1}\right) \quad \rightarrow \quad \left(\frac{P_2}{P_1}\right)^{1/(k-1)}\left(\frac{v_2}{v_1}\right)^{k/(k-1)} = 1$$

or

$$P_1 v_1^k = P_2 v_2^k \quad \rightarrow \quad P v^k = \text{constant}$$

i.e. another polytropic path, $P v^\gamma = \text{constant}$, with $\gamma = k = c_P/c_v$.



Remarks:

a) Same trend as isothermal but steeper because $k > 1$

b) True if c_v (or c_p) is constant (i.e. not a function of T), and true for ideal gases

c) c_p for monatomic ideal gas is approximately $(3R/2)$

- Work done by ideal gas under adiabatic expansion (with c_p constant)
  Then using $Pv^k = \text{constant} = P_i v_i^k = P_f v_f^k$

$$w_{by} = \int P\,dv = \int \frac{\text{constant}}{v^k} dv$$
$$= \frac{1}{1-k}\left(\frac{P_f v_f^k}{v_f^{k-1}} - \frac{P_i v_i^k}{v_i^{k-1}}\right) = \frac{1}{1-k}\left(P_f v_f - P_i v_i\right)$$

Or in terms of T,

$$w_{by} = \frac{R}{k-1}\left(T_i - T_f\right) = c_v\left(T_i - T_f\right) = -\Delta u$$

2. Open-system balances on process equipment (pp. 95-102)

a) Nozzles and diffusers
- Main function is to change velocities (i.e. $\Delta E_k \neq 0$)
- $Q_{in} = 0, W_{by,shaft} = 0, \Delta E_p \approx 0$
- → $h_{out} - h_{in} = e_{k,in} - e_{k,out}$

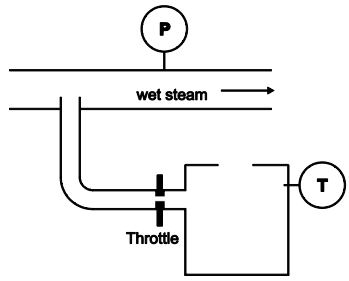
b) Turbines and pumps

(check out video on steam turbines (https://www.youtube.com/watch?v=MulWTBx3szc)
- Main function is to convert energy to work (turbines) or convert work to increase energy (pumps and compressors)
- $Q_{in} = 0, W_{by,shaft} \neq 0, \Delta E_p \approx 0$
- $(h_{out} - h_{in}) + \left(e_{k,out} - e_{k,in}\right) = -w_{by,shaft}$

c) Heat exchangers
- Main function is to transfer energy to change the temperature of feed
- $Q_{in} \neq 0, W_{by,shaft} = 0, \Delta E_p \approx 0, \Delta E_k \approx 0$
- $(h_{out} - h_{in}) = q_{in}$

d) Throttling devices
- Main function is to reduce pressure of feed
- $Q_{in} = 0, W_{by,shaft} = 0, \Delta E_p \approx 0, \Delta E_k \approx 0$
- $h_{out} = h_{in}$ (also known as "isenthalpic process")

Example: Throttle Calorimeter



Purpose: To estimate steam quality of main steam line

Available measurements: P_{main}, T_{cal}, P_{atm}

Data from steam tables: $\hat{h}_{vap,main}(P_{main})$, $\hat{h}_{liq,main}(P_{main})$,
$\hat{h}_{vap,sat}(P_{atm})$ $T_{sat}(P_{atm})$, $c_{p,vap}$

Energy balance: $\hat{h}_{main} = \hat{h}_{cal}$

$$x\,\hat{h}_{vap,main} + (1-x)\hat{h}_{liq,main} = \hat{h}_{vap,sat} + c_{p,vap}(T_{cal} - T_{sat})$$

$$x = \frac{\hat{h}_{vap,sat} - \hat{h}_{liq,main} + c_{p,vap}(T_{cal} - T_{sat})}{(\hat{h}_{vap,main} - \hat{h}_{liq,main})}$$

Remarks:

1. Care should be taken to not siphon condensed liquid in main line
2. Calculation might be improved by using steam table (functions) instead of c_p.

Assignment 2:

2.19, 2.24, 2.25, 2.30, 2.41, 2.51