CM 416 First Exam (Open-book open-notes)
January 5, 1995 7-9 pm
Name: ____________________

1. (20 pts) In terms of design parameters α, β and V, a second order system is modeled by the following equation

$$\frac{d^2C_a}{dt^2} + 2\alpha\frac{dC_a}{dt} + VC_a = \beta$$

Actual data from a run starting at $C_a(0) = 5$ and $dC_a/dt = 0$ is shown in Figure 1. Obtain the values of α, β and V using the data.

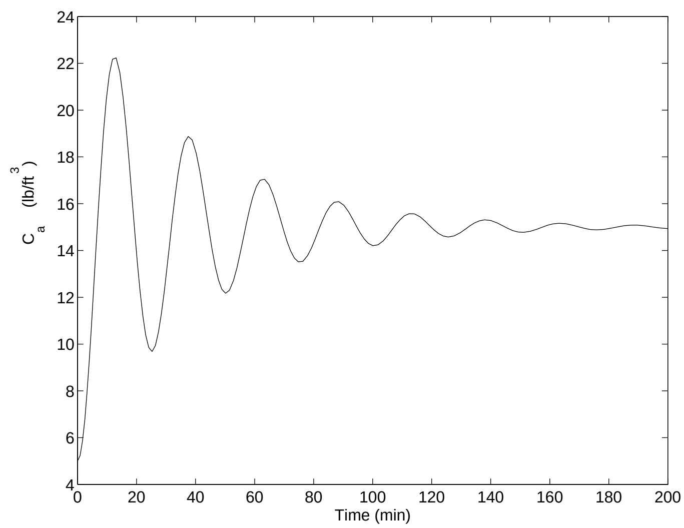

Figure 1: Second order system response.

2. (10 pts) Obtain a first order differential equation which would yield the data shown in Figure 2.

3. (10 pts) The pressure of a vessel can be described by

$$8\frac{d^2P}{dt^2} + 2\frac{dP}{dt} + 2P = 6$$

What is the damping coefficient of the system ? Is it underdamped or overdamped ?

4. Consider a tank whose cross-sectional area depends on height, $A = A(h)$. The differential volume at $h(t)$ is then given by $dV = A(h)dh$

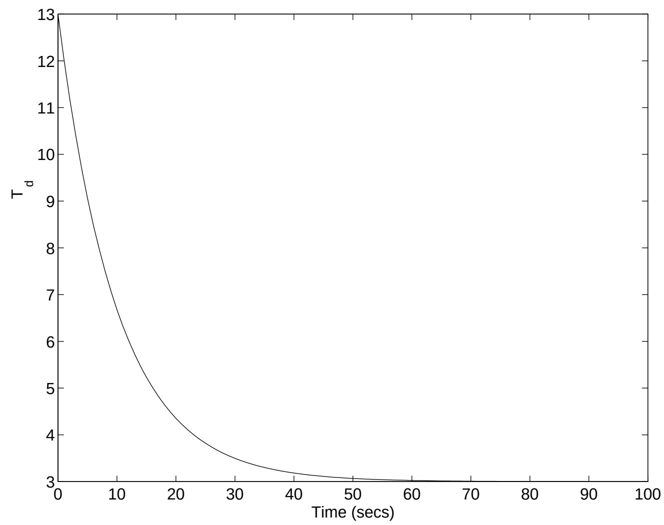

Figure 2: First order system response.

(a) (10 pts) Show that the process model for liquid height in the tank shown in Figure 3 is given by

$$\frac{dh}{dt} = \frac{F_{in} - k\sqrt{h}}{A(h)}$$

where F_{in} is the volumetric flow of liquid into the tank and the flow out of the tank is given by $k\sqrt{h}$.

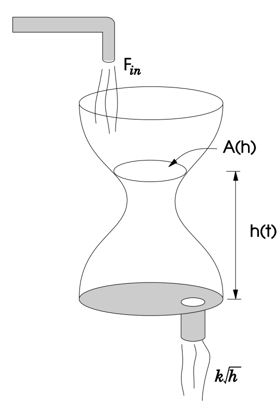

Figure 3: A tank where cross-sectional area depends on height.

(b) (5 pts) Derive a process model which describes the rate of change in height for a paraboloid tank shown in Figure 4 whose radius r changes with h according to $r = 2h^2$

5. The temperature in a reactor is described by the following nonlinear equation:

$$20\frac{dT}{dt} = \left[0.01T^2 - T + 20\right] - [0.5T - 30]$$

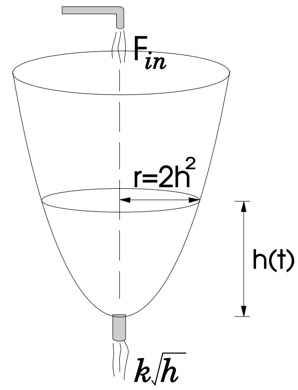

Figure 4: Spherical tank system.

(a) (10 pts) What are the steady state temperatures of this system ?

(b) (10 pts) Obtain a linearized equation for the system around the temperature $T = 100^o$F.

6. (15 pts) A two stirred tank system is shown in Figure 5 where a fraction α of the flow out of tank 2 is recycled into tank 1. Assuming that the liquid volumes in both tanks are kept constant, the temperature dynamics for both tanks are described by

$$\frac{dT_1}{dt} = \frac{F_o}{V_1}\left[(T_{in} - T_1) + \frac{\alpha}{1-\alpha}(T_2 - T_1)\right]$$
$$\frac{dT_2}{dt} = \frac{F_o}{V_2}\frac{1}{1-\alpha}(T_1 - T_2)$$

Let $F_o = 5$, $V_1 = 10$, $V_2 = 5$, $\alpha = 0.3$, obtain the characteristic equation and the eigenvalues for the system.

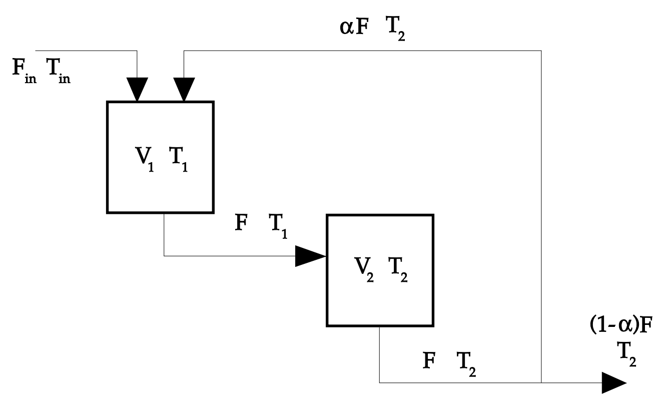

Figure 5: Two tank system with recycle.

7. (10 pts) For a temperature control system, the process is described by

$$\frac{dT'}{dt} = \frac{1}{\tau}(T' + u')$$

where T' is the deviation temperature and u is deviation manipulated variable. Show that using the proportional control law

$$u' = K_c(T'_{set} - T')$$

will yield a steady state offset: $T'_{ss} - T'_{set}$ that is not zero, for $K_c < \infty$, where T'_{ss} is the steady state temperature.

8. (Bonus: 5 pts) Describe briefly how the pressure regulator shown in Figure 6 is self-regulating for P_2, assuming $P_2 < P_1$ always.

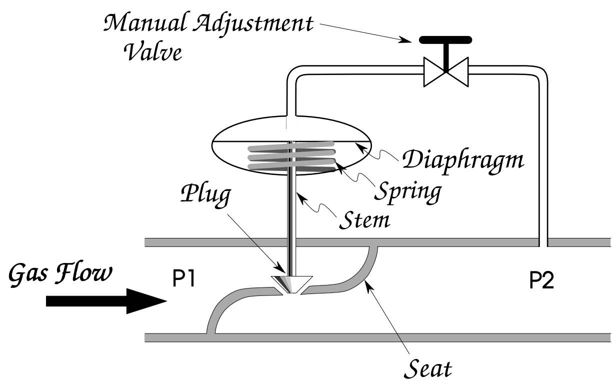

Figure 6: A pressure regulator system.