Solution to exam 1 dated January 5, 1995

1. Put the given model into the standard form,

$$\left(\frac{1}{V}\right)\cdot\frac{d^2}{dt^2}C_A + \left(\frac{2\cdot\alpha}{V}\right)\cdot\frac{d}{dt}C_A + C_A = \frac{\beta}{V}$$

Thus comparing to coefficients of the standard form,

$$\tau_n^{\,2} = \frac{1}{V}$$

$$2\cdot\tau_n\cdot\zeta = \frac{2\cdot\alpha}{V}$$

$$\frac{\beta}{V} = K_p$$

From the figure,

$$\text{Overshoot} := 0.75 \qquad 0.75 = \exp\left(-\pi\cdot\frac{\zeta}{\sqrt{1-\zeta^2}}\right) \qquad \zeta := 0.091$$

$$\text{Period} := 25 \qquad 25 = \frac{2\cdot\pi\cdot\tau_n}{\sqrt{1-\zeta^2}} \qquad \tau_n := 3.96$$

$$K_p := 15$$

so finally,

$$V := \frac{1}{\tau_n^{\,2}} \qquad V = 0.064$$

$$\alpha := \tau_n\cdot\zeta\cdot V \qquad \alpha = 0.023$$

$$\beta := K_p\cdot V \qquad \beta = 0.957$$

2. from the figure, $\tau = 10$

so the model is given by, $\frac{1}{10}\cdot\frac{d}{dt}T + T = 3$

3. In standard form, the system is given by

$$4\cdot\frac{d^2}{dt^2}P + \frac{d}{dt}P + P = 3$$

$\tau_n = 2 \qquad \zeta = \frac{1}{2\cdot\tau_n} = \frac{1}{4}$ so the system is underdamped

4. b) $\frac{d}{dt}h = \frac{F_{in} - k\cdot\sqrt{h}}{4\cdot\pi\cdot h^4}$

5. a) the steady states are given by,

$$0 = \left(0.01\cdot T_s^2 - T_s + 20\right) - \left(0.5\cdot T_s - 30\right)$$

$T_s := 50$ and $T_s := 100$

b) the linearized equation at T=100 is given by

$$\frac{d}{dt}T = \frac{1}{40}\cdot(T - 100)$$

6. $\frac{d}{dt}T_1 = \frac{1}{2}\cdot\left[\left(T_{in} - T_1\right) + 0.429\cdot\left(T_2 - T_1\right)\right]$

$$\frac{d}{dt}T_2 = 1.429\cdot\left(T_1 - T_2\right)$$

characteristic equation: $\det\left(\begin{bmatrix} s + \frac{1.429}{2} & -\frac{0.429}{2} \\ -1.429 & s + 1.429 \end{bmatrix}\right) = 0$

$$s^2 + 2.1435\cdot s + .7145 = 0$$

eigenvalues: $s = \begin{bmatrix} -.413 \\ -1.731 \end{bmatrix}$

7. $\frac{d}{dt}T = \frac{1}{\tau}\cdot\left[T + K_c\cdot\left(T_{set} - T\right)\right]$

at steady state, $T_{ss} + K_c\cdot\left(T_{set} - T_{ss}\right) = 0 \qquad T_{ss} = -K_c\cdot\frac{T_{set}}{\left(1 - K_c\right)}$

$$\text{Offset} = T_{ss} - T_{set} = -K_c\cdot\frac{T_{set}}{\left(1 - K_c\right)} - T_{set} = \frac{T_{set}}{K_c - 1}$$

thus $\text{Offset} \neq 0$ for $K_c < \infty$