Solution to Exam 1 dated December 14, 1995

1.
- V1 changes flow rate into tank 1 but can not affect temperature of cooling water inlet to tan k 1.
- V2 changes flow rate of cooling water to tank 1 but can not affect liquid level in tank 2.
- V3 is connected to a temperature indicator.
- V4 changes flow rate of cooling water to tank 2 but can not affect flow rate of liquid into tank 1
- V5 changes flow rate out of tank 2 but can not affect the liquid level of tank 1.

2. $$\frac{d}{dt}P_1 = \frac{R \cdot T_1}{V_1 \cdot MW} \cdot \left(k_1 \cdot \sqrt{P_0 - P_1} - k_2 \cdot \sqrt{P_1 - P_2}\right)$$

$$\frac{d}{dt}P_2 = \frac{R \cdot T_2}{V_2 \cdot MW} \cdot \left(k_2 \cdot \sqrt{P_1 - P_2} - k_3 \cdot \sqrt{P_2 - P_3}\right)$$

At steady state:

$$k_1 \cdot \sqrt{P_0 - P_{1s}} - k_2 \cdot \sqrt{P_{1s} - P_{2s}} = 0$$

$$k_2 \cdot \sqrt{P_{1s} - P_{2s}} - k_3 \cdot \sqrt{P_{2s} - P_3} = 0$$

Solving simultaneously,

$$P_{1s} = \frac{k_3^2 \cdot k_2^2}{\left(k_1^2 \cdot k_2^2 + k_2^2 \cdot k_3^2 + k_1^2 \cdot k_3^2\right)} \cdot P_3 + \frac{\left(k_1^2 \cdot k_2^2 + k_1^2 \cdot k_3^2\right)}{\left(k_1^2 \cdot k_2^2 + k_2^2 \cdot k_3^2 + k_1^2 \cdot k_3^2\right)} \cdot P_0$$

$$P_{2s} = \frac{\left(k_2^2 \cdot k_3^2 + k_1^2 \cdot k_3^2\right)}{\left(k_1^2 \cdot k_2^2 + k_2^2 \cdot k_3^2 + k_1^2 \cdot k_3^2\right)} \cdot P_3 + \frac{\left(k_1^2 \cdot k_2^2\right)}{k_1^2 \cdot k_2^2 + k_3^2 \cdot \left(k_2^2 + k_1^2\right)} \cdot P_0$$

3. First obtain the steady states,

$$0=\frac{9}{100}-\frac{1}{10}\cdot x_s-2\cdot x_s\cdot y_s$$

$$0=\frac{1}{100}-\frac{3}{5}\cdot y_s+2\cdot x_s\cdot y_s$$

linearizing around the steady state,

$$\frac{d}{dt}x=\frac{1}{10}\cdot\left(x_{in}-\frac{9}{10}\right)-\left(\frac{1}{10}+2\cdot y_s\right)\cdot\left(x-x_s\right)-\left(2\cdot x_s\right)\cdot\left(y-y_s\right)$$

$$\frac{d}{dt}y=\frac{1}{10}\cdot\left(y_{in}-\frac{1}{10}\right)+\left(2\cdot y_s\right)\cdot\left(x-x_s\right)-\left(\frac{3}{5}-2\cdot x_s\right)\cdot\left(y-y_s\right)$$

Solving simultaneously, there are 2 steady states. One at:

$x_s=0.259$ $\quad$ $y_s=0.123$

and the other at

$x_s=1.041$ $\quad$ $y_s=-6.752\cdot 10^{-3}$

Around the first steady state,

$$\frac{d}{dt}x=0.1\cdot\left(x_{in}-0.9\right)-(0.346)\cdot(x-0.259)-(0.518)\cdot(y-0.123)$$

$$\frac{d}{dt}y=0.1\cdot\left(y_{in}-0.1\right)+(0.246)\cdot(x-0.259)-(0.082)\cdot(y-0.123)$$

while around the second steady state,

$$\frac{d}{dt}x=0.1\cdot\left(x_{in}-0.9\right)-(0.087)\cdot(x-1.04)-(2.08)\cdot\left[y-\left(-6.75\cdot 10^{-3}\right)\right]$$

$$\frac{d}{dt}y=0.1\cdot\left(y_{in}-0.1\right)+(-0.014)\cdot(x-1.04)-(-1.48)\cdot\left[y-\left(-6.75\cdot 10^{-3}\right)\right]$$