Solution to First Exam December 19, 1996

1. $$\frac{d}{dt}T = -3\cdot T + \frac{1}{2}\cdot C + 1$$

$$\frac{d}{dt}C = T - 5\cdot C + 2$$

Characteristic Equation: $$(s+5)\cdot(s+3) - \frac{1}{2} = 0$$

Eigenvalues: $$s = \begin{bmatrix} -4 + \frac{1}{2}\cdot\sqrt{6} \\ -4 - \frac{1}{2}\cdot\sqrt{6} \end{bmatrix}$$

2. Overshoot = $$\frac{29}{55} = \exp\left(-\pi\cdot\frac{\zeta}{\sqrt{1-\zeta^2}}\right)$$

$$\zeta = \frac{-\ln\left(\frac{29}{55}\right)}{\sqrt{\ln\left(\frac{29}{55}\right)^2 + \pi^2}} \qquad \frac{-\ln\left(\frac{29}{55}\right)}{\sqrt{\ln\left(\frac{29}{55}\right)^2 + \pi^2}} = 0.2$$

$$T := 120 \qquad 120 = 2\cdot\pi\cdot\frac{\tau}{\sqrt{1-\left(\frac{2}{10}\right)^2}} \qquad \tau := 18.7$$

$$18.7^2\cdot\frac{d^2}{dt^2}x + 2\cdot 18.7\cdot 0.2\cdot\left(\frac{d}{dt}x\right) + x = 100$$

3. $\frac{d}{dt}x - (2\cdot k - 4)\cdot x = 3$ eigenvalue $s = 2\cdot k - 4$

$k \geq 2$ for instability

4. $\frac{d}{dt}x = 0.414 + -(x - 1) - 2\cdot(y - 1) + 1.414\cdot(u - 0.5)$

$\frac{d}{dt}y = 2.5 + 6\cdot(x - 1) - 0.5\cdot(y - 1) - (u - 0.5)$

5. $\frac{d}{dt}h_1 = \frac{\left(F_o + \alpha\cdot k_2\cdot\sqrt{h_2} - k_1\cdot\sqrt{h_1}\right)}{A}$

$\frac{d}{dt}h_2 = \frac{\left(k_1\cdot\sqrt{h_1} - k_2\cdot\sqrt{h_2}\right)}{1.5\cdot A}$

steady states: $h_{1s} = \left(\frac{k_2}{k_1}\right)^2\cdot h_{2s}$

$$F_o + \alpha\cdot k_2\cdot\sqrt{h_{2s}} - k_1\cdot\sqrt{\left(\frac{k_2}{k_1}\right)^2\cdot h_{2s}} = 0$$

$$h_{2s} = \begin{bmatrix} \dfrac{F_o^2}{\left[k_2^2\cdot(\alpha + 1)^2\right]} \\ \dfrac{F_o^2}{\left[k_2^2\cdot(\alpha - 1)^2\right]} \end{bmatrix} \qquad h_{1s} = \begin{bmatrix} \dfrac{\left(F_o\cdot\frac{k_2}{k_1}\right)^2}{\left[k_2^2\cdot(\alpha + 1)^2\right]} \\ \dfrac{\left(F_o\cdot\frac{k_2}{k_1}\right)^2}{\left[k_2^2\cdot(\alpha - 1)^2\right]} \end{bmatrix}$$