# Solution to CM416 Exam 1, Dec. 16, 1998

1. @ t=10, x=8.13

$$\frac{8.13 - 5}{K_p - 5} = 0.632$$

$$K_p := 9.95$$

2. a)

$$x_{ss} = k_c \cdot (x_{set} - x_{ss})$$

$$x_{ss} = k_c \cdot \frac{x_{set}}{(1 + k_c)}$$

b) in standard form,

$$\left(\frac{1}{1 + k_c}\right) \cdot \frac{d^2}{dt^2} x + \left(\frac{2}{1 + k_c}\right) \cdot \frac{d}{dt} x + x = \frac{k_c}{1 + k_c} \cdot x_{set}$$

and thus,

$$\tau_n = \frac{1}{\sqrt{1 + k_c}} \qquad \zeta = \frac{\sqrt{1 + k_c}}{1 + k_c} = \frac{1}{\sqrt{1 + k_c}}$$

3. operating points: $C := 0.6$ $\quad C_{in} := 0.9$ $\quad T := 680$ $\quad T_{in} := 500$

parameters: $\alpha := 2$ $\quad \beta := -0.01$ $\quad k := 1$ $\quad \gamma := 600$

$$f_1 := \alpha \cdot (C_{in} - C) - k \cdot \exp\left(\frac{\beta}{T}\right) \cdot C$$

$$f_2 := \alpha \cdot (T_{in} - T) + \gamma \cdot \exp\left(\frac{\beta}{T}\right) \cdot C$$

$$f_1 = 8.823 \cdot 10^{-6}$$

$q_{cin} := \alpha$ $q_{cin} = 2$

$q_c := -\alpha - k \cdot \exp\left(\frac{\beta}{T}\right)$ $q_c = -3$

$q_T := k \cdot \frac{\beta}{T^2} \cdot \exp\left(\frac{\beta}{T}\right) \cdot C$ $q_T = -1.298 \cdot 10^{-8}$

$$\frac{d}{dt}C = 2 \cdot \left(C_{in} - 0.9\right) - 3 \cdot (C - 0.6)$$

$f_2 = -5.294 \cdot 10^{-3}$

$p_{Tin} := \alpha$ $p_{Tin} = 2$

$p_c := \gamma \cdot \exp\left(\frac{\beta}{T}\right)$ $p_c = 599.991$

$p_T := -\alpha - \gamma \cdot \frac{\beta}{T^2} \cdot \exp\left(\frac{\beta}{T}\right) \cdot C$ $p_T = -2$

$$\frac{d}{dt}T = 2 \cdot \left(T_{in} - 500\right) - 2 \cdot (T - 680) + 600 \cdot (C - 0.6)$$

4. Characteristic Equation: $\det\begin{bmatrix} s+1 & -4+K \\ -2 & s+2 \end{bmatrix} = 0$

$$s^2 + 3 \cdot s - 6 + 2 \cdot K = 0$$

Eigenvalues: $\begin{bmatrix} \frac{-3}{2} + \frac{1}{2} \cdot \sqrt{33 - 8 \cdot K} \\ \frac{-3}{2} - \frac{1}{2} \cdot \sqrt{33 - 8 \cdot K} \end{bmatrix}$

For critical stability: $\frac{-3}{2} + \frac{1}{2} \cdot \sqrt{33 - 8 \cdot K} = 0$ or $K := 3$