CM3310 Spring 2008

(Dr. Tom Co, 1/23/2008)

Lecture 3. Process Modeling, part 2

Typical analysis of process models:

1. Steady states
2. Stability and sensitivity
3. Empirical analysis
4. Simulations

Steady state analysis:

1. Set all time derivatives to zero
2. Replace all process variables (states), manipulated variables and disturbances with their steady states, i.e. replace P with P_{ss}, etc.
3. Solve for the steady states in terms of the steady state manipulated and disturbance variables.

Example: Spherical surge tank

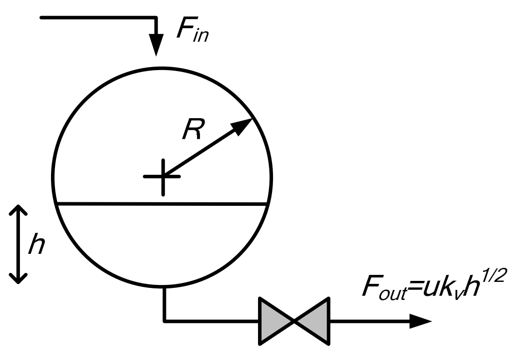

Figure 1.

$$\frac{dh}{dt}=\frac{F_{in}-u\,k_v\sqrt{h}}{\pi\,h(2R-h)}$$

First, set $u=u_{ss}$ and $F_{in}=F_{in_{ss}}$, then set the time derivative to zero. Replace h with h_{ss}, then solve:

$$h_{ss}=\left(\frac{F_{in_{ss}}}{u_{ss}k_v}\right)^2$$

Note: the steady state is independent of the radius *R*.

Remarks:

1. Processes can have multiple steady states
2. The roots need to be real values.
3. Usually need numerical methods to find roots

Linearization: (page 60-66)

1. A local analysis around a chosen operating point (usually the steady state).
2. Approximates nonlinear equations with linear equations.
3. The linearized equations can be used to predict stability of steady states.
4. The coefficients of linearized equations also yield sensitivity analysis.

Method: (use Taylor series)

Given:

$$\frac{dx}{dt} = f(x, u, d)$$

And operating point: (x_{op}, u_{op}, d_{op})

Required: approximate linear equation of the form

$$\frac{dx}{dt} = \alpha(x - x_{op}) + \beta(u - u_{op}) + \gamma(d - d_{op}) + \delta$$

Calculations:

$$\delta = f|_{x=x_{op},u=u_{op},d=d_{op}}$$
$$\alpha = \left.\frac{\partial f}{\partial x}\right|_{x=x_{op},u=u_{op},d=d_{op}}$$
$$\beta = \left.\frac{\partial f}{\partial u}\right|_{x=x_{op},u=u_{op},d=d_{op}}$$
$$\gamma = \left.\frac{\partial f}{\partial d}\right|_{x=x_{op},u=u_{op},d=d_{op}}$$

Example 2.4 (page 62-63) Second order reaction

$$\frac{dC_A}{dt} = \frac{F}{V}(C_{Ain} - C_A) - kC_A{}^2$$

Linearize around steady state:

$$\frac{dC_A}{dt} = \left(-\frac{F_s}{V} - 2kC_{As}\right)(C_A - C_{As}) + \left(\frac{C_{Ain,s} - C_{As}}{V}\right)(F - F_s) + \left(\frac{F_s}{V}\right)(C_{Ain} - C_{Ains})$$

Stability Analysis of Linear Systems:

1. First, determine the eigenvalues of the system.
2. If any eigenvalue has a positive real part, the system is unstable.
3. The imaginary parts determine the frequency of oscillations.

(details on how to find eigenvalues will be given in the next lecture)