CM3310 Spring 2008

(Dr. Tom Co, 2/25/2008)

Lecture 12. Transfer Functions 2

1. Simple Feedback Loop

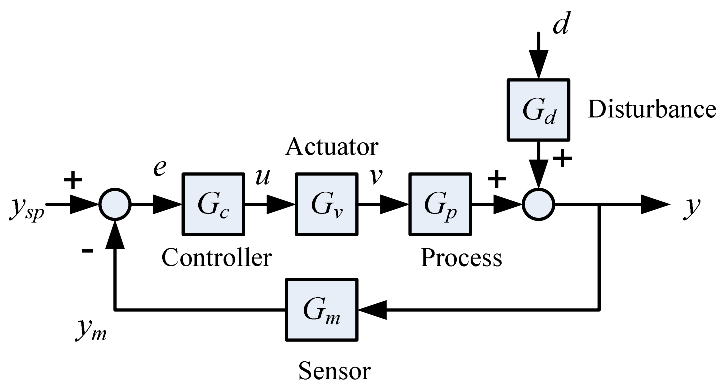

Figure 1. Feedback Control Block Diagram

$$\hat{y} = \frac{G_c G_v G_p}{1 + G_c G_v G_p G_m} \hat{y}_{sp} + \frac{G_d}{1 + G_c G_v G_p G_m} \hat{d}$$

Define

$G_{CL}(s)$ as the closed-loop transfer function (from setpoint to output)
$G_{DCL}(s)$ as the disturbance closed-loop transfer function (from disturbance to output).

Thus, in the example above we have

$$G_{CL}(s) = \frac{G_c G_v G_p}{1 + G_c G_v G_p G_m}$$

$$G_{DCL}(s) = \frac{G_d}{1 + G_c G_v G_p G_m}$$

Example:

$G_p = \frac{-s+1}{s^2+4s+3}$	$G_m = \frac{1}{2s+1}$

$G_v = \frac{3}{s^2+4s+4}$	$G_c = K_c \frac{\tau_{int}s+1}{\tau_{int}s}$
$G_d = \frac{2}{s^2+4s+3}$	$K_c = 1;\ \tau_{int} = 10$

Then,

$$G_{CL}(s) = \frac{-3s^3 + \frac{6}{5}s^2 + \frac{33}{20}s + \frac{3}{20}}{s^6 + \frac{17}{2}s^5 + 27s^4 + \frac{79}{2}s^3 + \frac{49}{2}s^2 + \frac{147}{20}s + \frac{3}{20}}$$

$$G_{DCL}(s) = \frac{2s^4 + 29s^3 + 12s^2 + 4s}{s^6 + \frac{17}{2}s^5 + 27s^4 + \frac{79}{2}s^3 + \frac{49}{2}s^2 + \frac{147}{20}s + \frac{3}{20}}$$

Note: both $G_{CL}(s)$ and $G_{DCL}(s)$ have the same denominator, i.e. they will have the same roots.

a) Stability and performance analysis:
Poles of $G_{CL}(s)$ and $G_{DCL}(s)$:
$-3.4371, -2.0837 \pm 1.069i, -0.4368 \pm 0.4142i, -0.0220$

➔ Stable
➔ exhibits oscillations
➔ slowest mode has time constant of approximately $0.0220^{-1} = 45\ seconds$.

b) Steady state analysis:

Let $y_{sp}(t) = Step(t)$ and $d(t) = 0.5\ Step(t)$. Then the Laplace transforms are given by

$$\hat{y}_{sp} = \frac{1}{s} \quad and \quad \hat{d} = 0.5\ \frac{1}{s}$$

Substituting, we find

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} s\ \hat{y} = \lim_{s\to 0} s\ G_{CL}\hat{y}_{sp} \quad + \lim_{s\to 0} s\ G_{DCL}\hat{d}$$

$$= \lim_{s\to 0} s \left(\frac{-3s^3 + \frac{6}{5}s^2 + \frac{33}{20}s + \frac{3}{20}}{s^6 + \frac{17}{2}s^5 + 27s^4 + \frac{79}{2}s^3 + \frac{49}{2}s^2 + \frac{147}{20}s + \frac{3}{20}} \right) \left(\frac{1}{s}\right)$$

$$+\lim_{s\to 0} s\left(\frac{2s^4+29s^3+12s^2+4s}{s^6+\frac{17}{2}s^5+27s^4+\frac{79}{2}s^3+\frac{49}{2}s^2+\frac{147}{20}s+\frac{3}{20}}\right)\left(\frac{0.5}{s}\right)$$

= 1

Thus, offset is zero.

Issue: What if we want to investigate different values of control tunings ?

Approaches:

a) We could plot the poles and see where they move as we vary the controller tuning values → we obtain a root locus diagram.

b) For stability, we could use a quicker method to determine whether the chosen controller parameters will be unstable → Routh Hurwitz method. (see link in http://www.chem.mtu.edu/~tbco/cm416/routh.html)