CM3310 Spring 2008

(Dr. Tom Co, 3/26/2008)

Lecture 18. Nyquist Stability Criterion

1. Recall Connections between Nyquist Plots and Transfer Functions

$$ReG = Real\big(\,G(i\omega)\big) \quad \text{and} \quad ImG = Imag\big(\,G(i\omega)\big)$$

2. Main Result: Nyquist Stability Criterion

For the feedback loop shown in Figure 1,

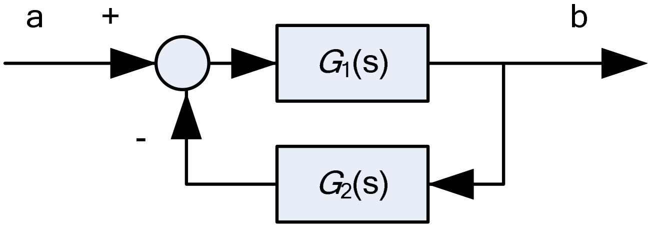


Figure 1.

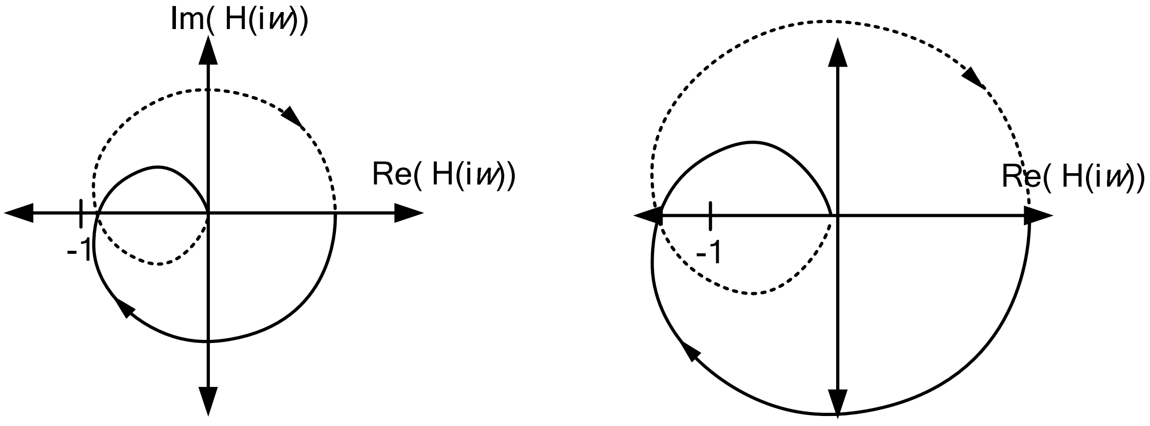
let the return loop is given by

$$H(s) = G_1(s)G_2(s)$$

be assumed stable. Then the closed-loop transfer function given by

$$G_{CL}(s) = \frac{G_1(s)}{1 + H(s)}$$

is asymptotically stable if the Nyquist plot of $H(s)$ does not encircle the point $(-1,0)$.



stable

unstable

Figure 2.

Development of the Criterion:

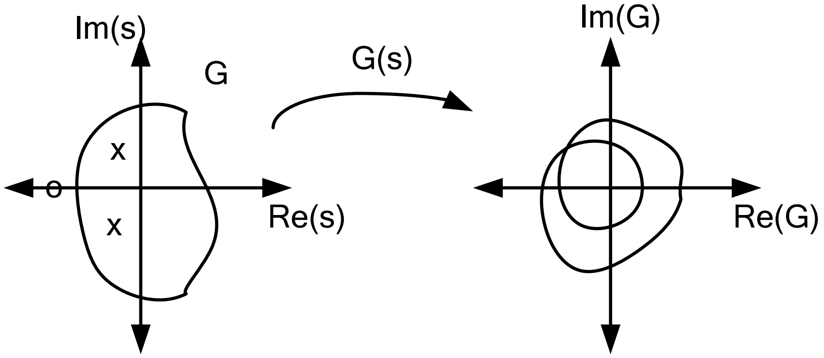
1. Complex Mapping Theorem:

Given: a transfer function $G(s)$ and simple closed contour Γ,

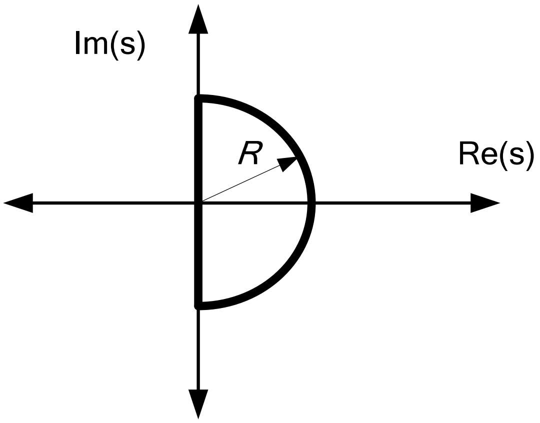
$$G(s) = \frac{num(s)}{den(s)} = \frac{b_{n-1}s^{n-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}$$

Let Z be the number of zeroes of $G(s)$ inside Γ and let P be the number of poles of $G(s)$ inside Γ.

Then as s traverse the contour given by Γ in the clockwise manner, the map of $G(\mathrm{s})$ will encircle the origin $N = Z - P$ times in the clockwise manner (where $N < 0$ means counterclockwise encirclements).



2. Nyquist Contour



3. Stability Criterion

a) Physically realizable transfer functions are "proper", i.e. the order of the denominator is greater or equal to the order of the numerator, and

$$\lim_{s \to \infty} H(s) = \alpha < \infty$$

This means at the radius of the Nyquist contour goes to infinity, the map is just a constant. (Note: if $\alpha = 0$, then the process is "strictly proper".)

b) The frequency response data gives the upper branch map $H(i\omega)$. This means that as $s = i\omega$, with $0 < \omega < \infty$, the map $H(s)$is given by the Nyquist plot of the frequency response data.

c) The map $H(-i\omega)$ will yield a curve that is a vertical mirror image of $H(i\omega)$.

d) Thus, the Nyquist plot, together with complex mapping theorem implies that the closed loop system is stable if the Nyquist plot of $\left(1 + H(s)\right)$ does not encircle the origin. Equivalently, for stability, we need the Nyquist plot of $H(s)$ not to encircle the point $(-1,0)$.

3. Stability Margins (see page 231-234) – as a measure of feedback stability robustness

a) Gain Margin

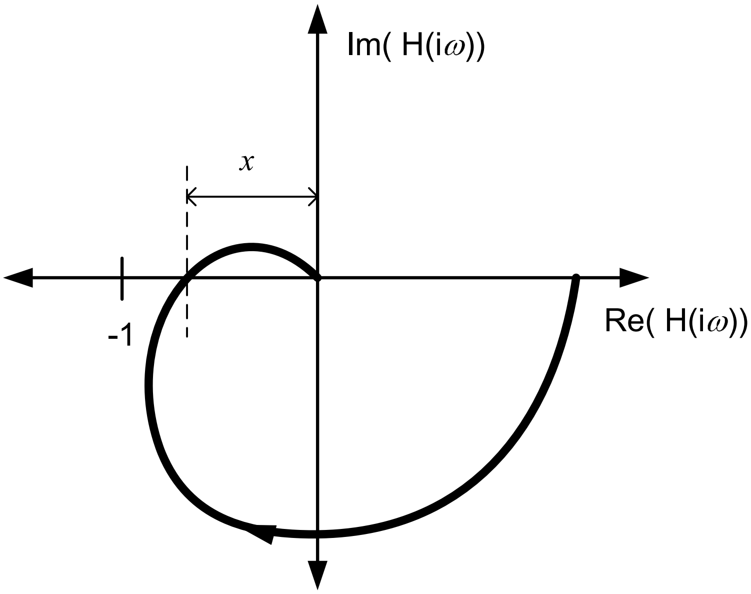
$$GM = \frac{1}{x}$$

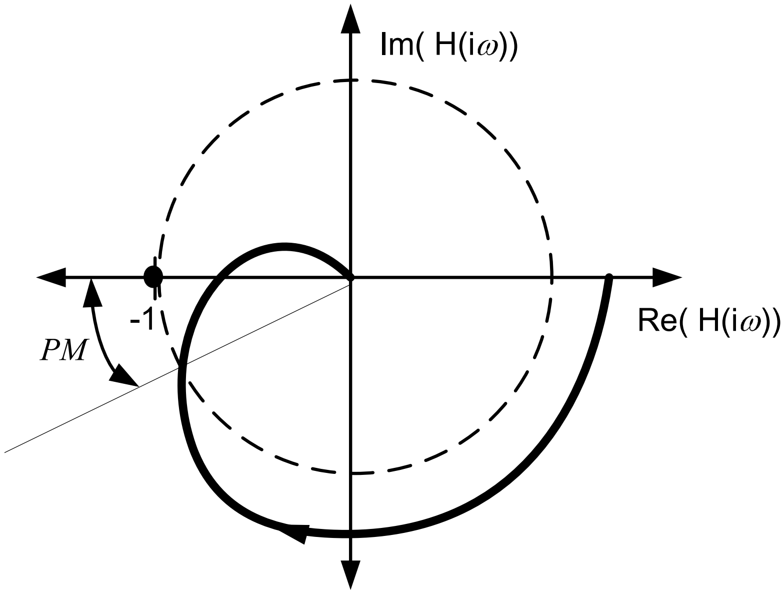
where x is the distance from the origin to the intersection of the Nyquist plot with the negative real line.

b) Phase Margin

$$PM = 180° + \phi_{[1]}$$

where $\phi_{[1]} < 0$ is the phase shift when the Nyquist plot intersects the unit circle.

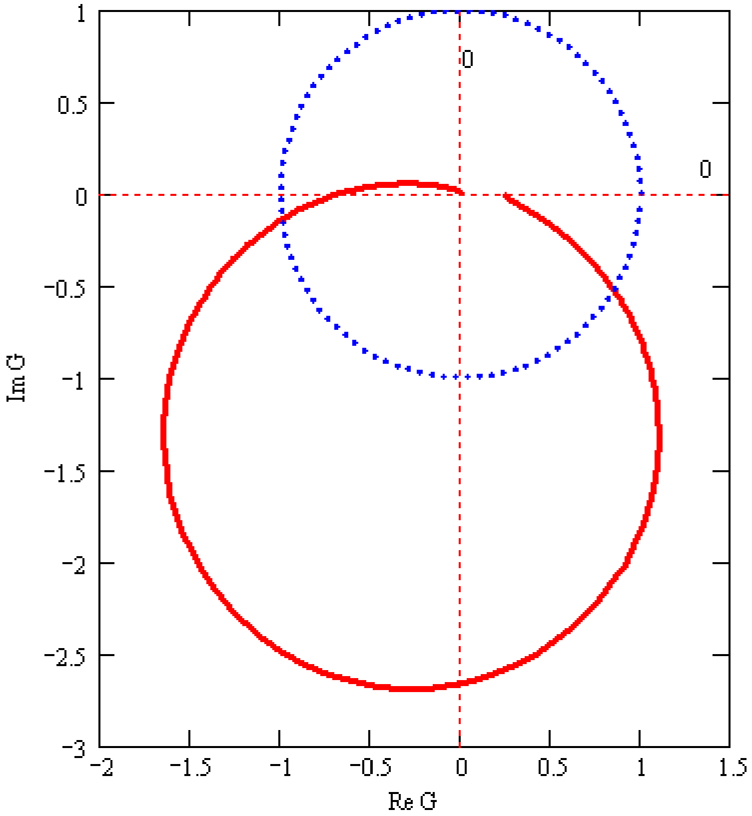

Im(H(iω))
x
-1
Re(H(iω))


Im(H(iω))
-1
PM
Re(H(iω))

Example:

$$H(s) = \frac{0.25(-0.08s + 1)}{(0.2s + 1)((1.1s)^2 + 0.1s + 1)}$$

The Nyquist plot is given by



From the figure, one can measure $x = 0.67$ and $\phi_{[1]} = -172°$. Thus we have

$$GM = \frac{1}{0.67} = 1.49 \qquad PM = 180° - 172° = 8°$$