CM3310 Spring 2008

(Dr. Tom Co, 3/31/2008)

Lecture 19. Nyquist Plots For PID Tuning

1. Effects on Nyquist plots by Adding Proportional Control Gain

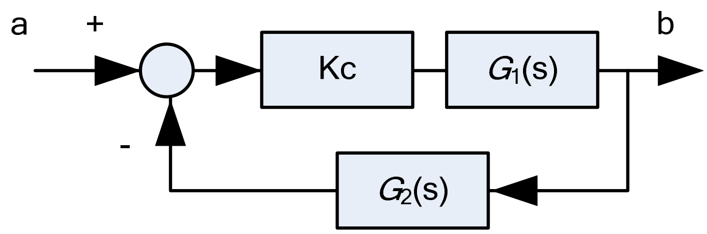

Figure 1.

then the new return loop is given by

$$M(s) = K_c H(s) \quad \text{where} \quad H(s) = G_1(s) G_2(s)$$

where the closed-loop transfer function is given by

$$G_{CL}(s) = \frac{K_c G_1(s)}{1 + M(s)} = \frac{K_c G_1(s)}{1 + K_c H(s)}$$

The Nyquist plot values of $M(s)$ is given by

$$ReM = \mathrm{Re}\big(K_c\, H(i\omega)\big) = K_c\, \mathrm{Re}\big(K_c\, H(i\omega)\big)$$
$$ImM = \mathrm{Im}\big(K_c\, H(i\omega)\big) = K_c\, \mathrm{Im}\big(K_c\, H(i\omega)\big)$$

Bottom Line:

1. If $|K_c| > 1$, the gain will magnify the Nyquist plot of $H(s)$ proportionally. Otherwise, if $|K_c| < 1$, the gain will reduce the Nyquist plot of $H(s)$ proportionally.
2. If $K_c < 0$, the Nyquist plot of $H(s)$ will be flipped horizontally along the imaginary line and flipped vertically along the real line.

Example:

$$H(s) = \frac{1}{(s+1)^4}$$

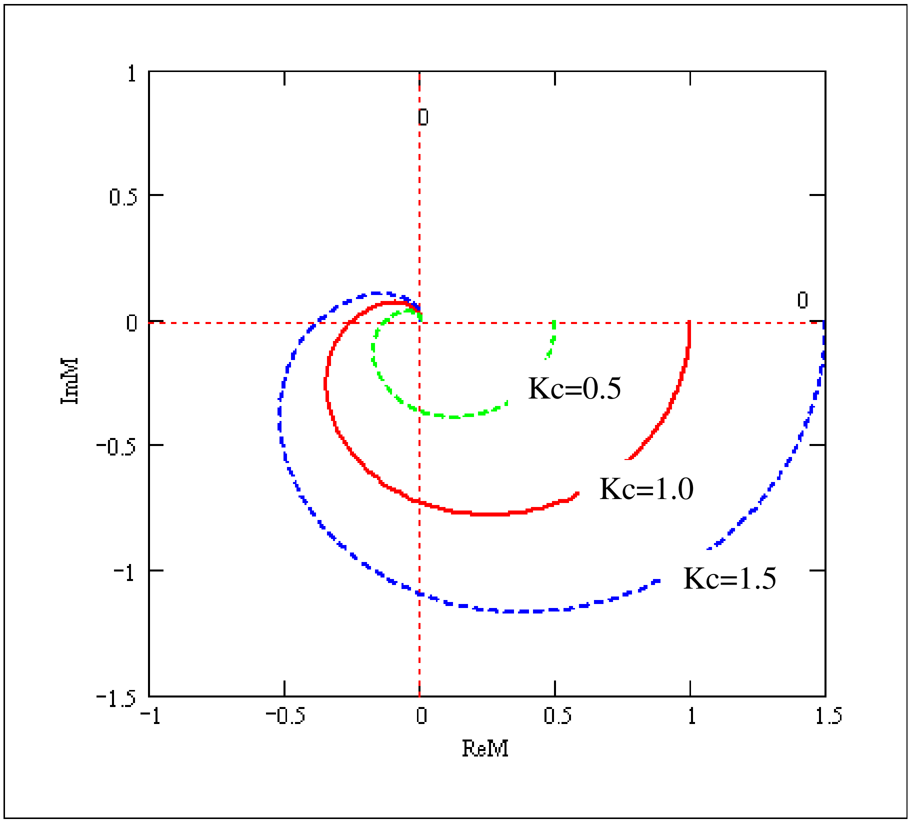

Figure 2.

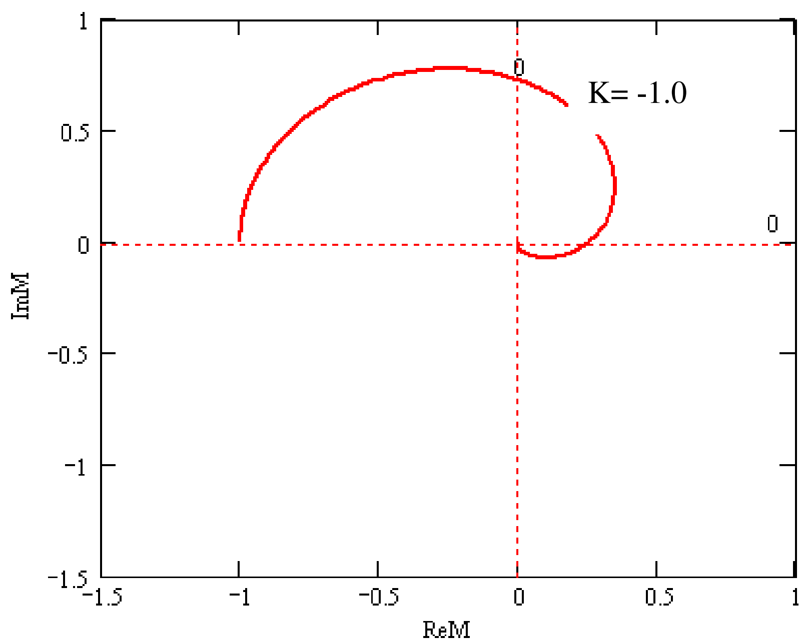

Figure 3.

2. Implications for Ziegler-Nichols Tuning:

a. The ultimate gain, K_u, is the vale of the gain such that the Nyquist plot of $M(s)$ passes through the critical point $(-1,0)$.
b. To determine the ultimate period, one needs to go back to the data and determine the frequency when the imaginary part is zero and the real part is negative.
c. The prescribed value for proportional control is $K_c = 0.5\ K_u$ is to yield a gain margin of 2.

Example: (continuation of previous example, with frequency in rads/sec)

$$H(i\omega) = \alpha(\omega) + i\beta(\omega)$$

where,

$$\alpha(\omega) = \frac{\omega^4 - 6\omega^2 + 1}{(\omega^4 - 6\omega^2 + 1)^2 + (-4\omega^3 + 4\omega)^2}$$
$$\beta(\omega) = \frac{4\omega^3 - 4\omega}{(\omega^4 - 6\omega^2 + 1)^2 + (-4\omega^3 + 4\omega)^2}$$

Solving for ω_u such that $\beta(\omega_u) = 0$, we find

$$\omega_u^2 = 1 \quad \text{or} \quad \omega_u = \pm 1$$

With $\alpha(\omega_u) = -0.25$, the ultimate gain is given by

$$K_u = \frac{1}{|-0.25|} = 4$$

The ultimate period is

$$P_u = \frac{2\pi}{\omega_u} = 6.28 \text{ sec}$$

For a PID controller based on Ziegler-Nichols tuning we evaluate

$$K_c = \frac{K_u}{1.7} = 2.35$$
$$\tau_{\text{int}} = \frac{P_u}{2} = 3.14$$
$$\tau_{\text{der}} = \frac{P_u}{8} = 0.785$$