Tutorial on Optimizing Control Tuning Using Euler Method and SOLVER

(Tom Co, 2/22/2008)

Given

Process:

$$\frac{dz_1}{dt} = -3z_1 + z_2 - 0.1u$$
$$\frac{dz_2}{dt} = -3z_1 + 2u$$
$$z_1(0) = z_2(0) = 0$$
$$y = z_1$$

PI Control:

$$u = K_c\left(e + \frac{1}{\tau_{int}}\int e\,dt\right)$$

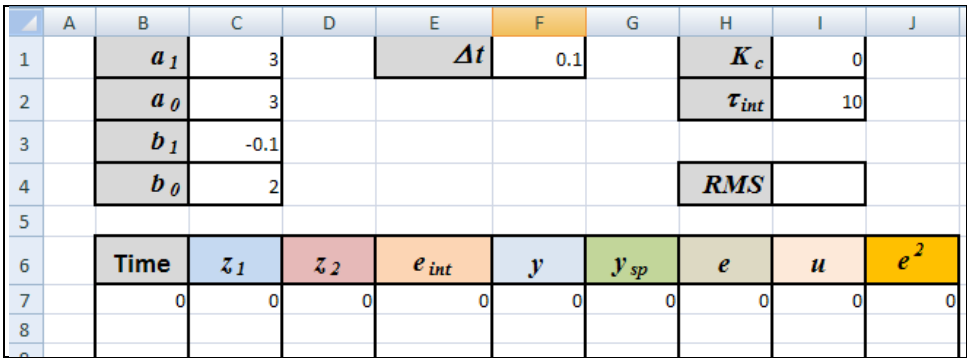
$$e = y_{sp} - y$$

Set Point:

$$y_{sp} = \begin{cases} 0 & if \;\; t \le 5 \\ 1 & if \;\; t > 5 \end{cases}$$

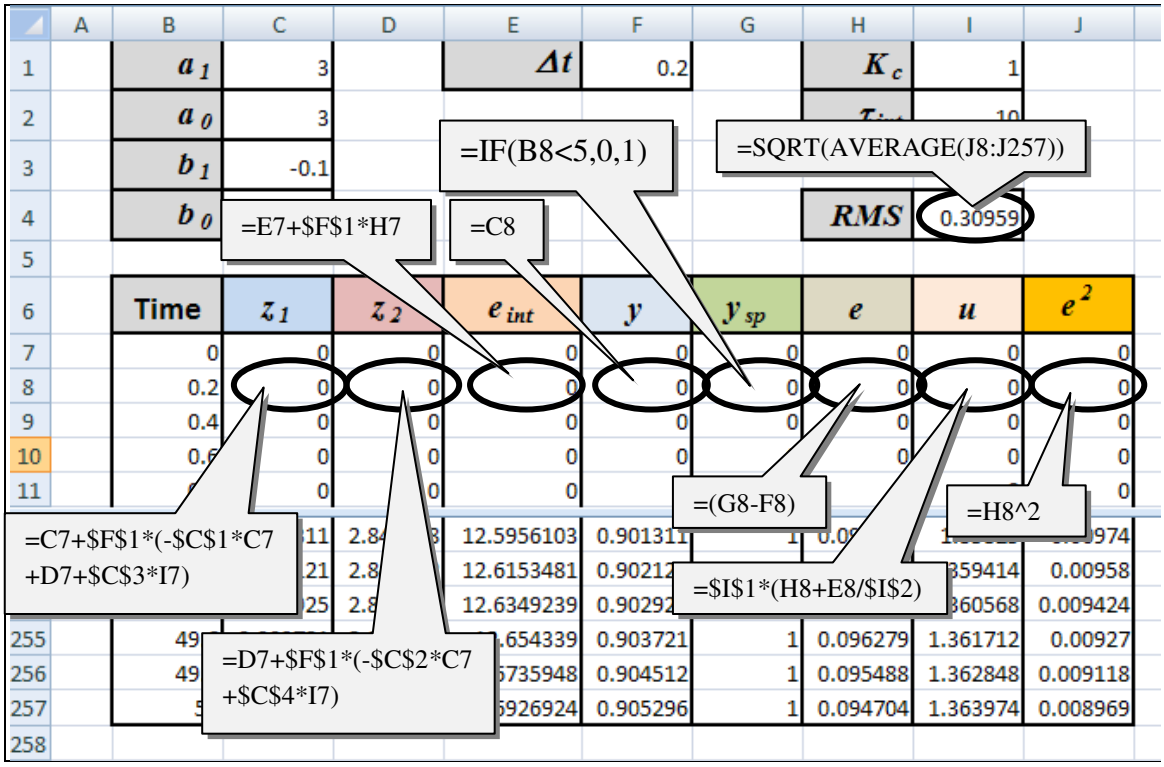
Procedure

1. Let $e_{int} = \int e\,dt$. Then introduce an additional differential equation
$$\frac{de_{int}}{dt} = e \quad ; \quad e_{int}(0) = 0$$
2. The PI control then becomes
$$u = K_c\left(e + \frac{1}{\tau_{int}}e_{int}\right)$$
3. Prepare an Excel spreadsheet, including the initial conditions as shown in Figure 1.
4. Include the formulas for numerical integration as shown in Figure 2.
5. Use solver to minimize the RMS error by adjusting K_c and τ_{int}. (see Figure 3).
6. Plot the controlled response together with the setpoint trajectory to check for acceptability. (see Figure 4.)



	A	B	C	D	E	F	G	H	I	J
1		a_1	3		Δt	0.1		K_c	0	
2		a_0	3					τ_{int}	10	
3		b_1	-0.1							
4		b_0	2					RMS		
5										
6		Time	z_1	z_2	e_{int}	y	y_{sp}	e	u	e^2
7		0	0	0	0	0	0	0	0	0
8										

Figure 1.



	A	B	C	D	E	F	G	H	I	J
1		a_1	3		Δt	0.2		K_c	1	
2		a_0	3						10	
3		b_1	-0.1							
4		b_0						RMS	0.30959	
5										
6		Time	z_1	z_2	e_{int}	y	y_{sp}	e	u	e^2
7		0	0	0	0	0	0	0	0	0
8		0.2	0	0	0	0	0	0	0	0
9		0.4	0	0	0	0	0	0	0	0
10		0.6	0	0	0	0	0	0	0	0
11			0	0	0					0
255		49			.654339	0.903721	1	0.096279	1.361712	0.00927
256		49			5735948	0.904512	1	0.095488	1.362848	0.009118
257		5			5926924	0.905296	1	0.094704	1.363974	0.008969
258										

Figure 2.

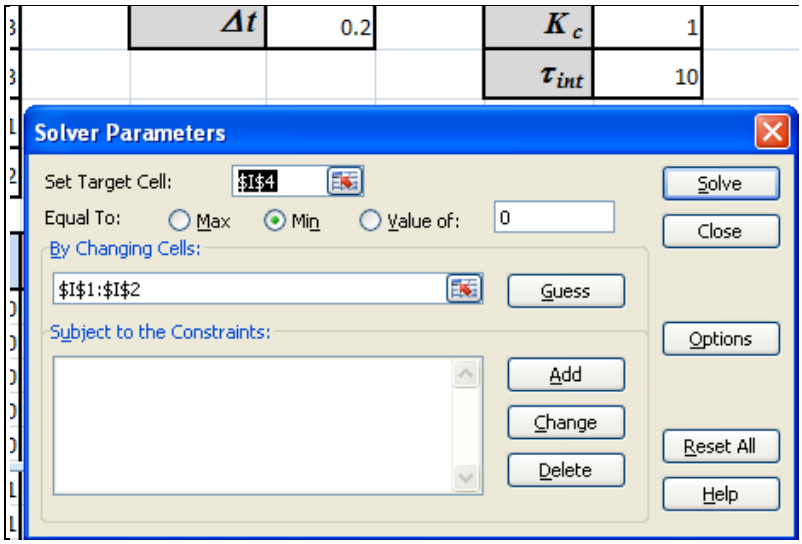


Figure 3.

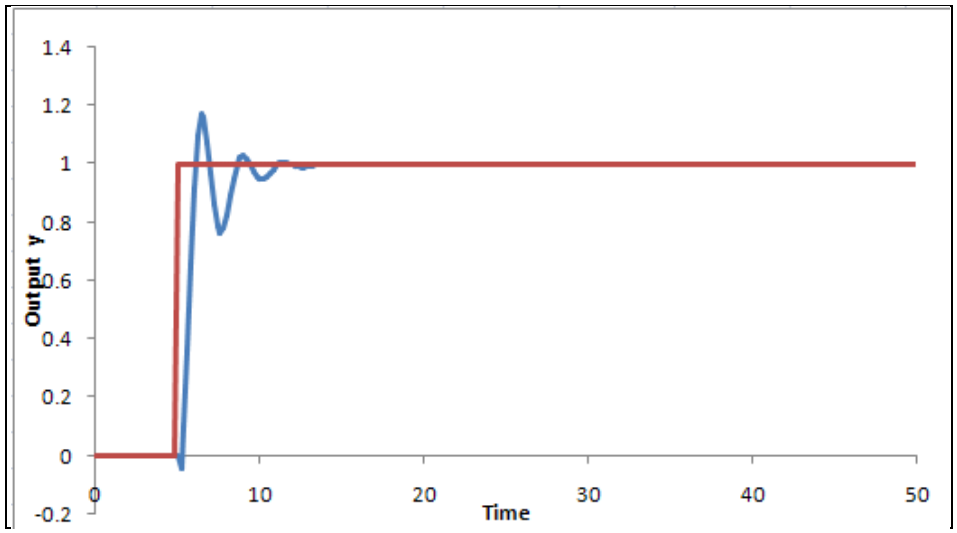


Figure 4.