First Exam. CM416
December 19, 1996 (6:30-8:30pm)
Open-Books/Open-Notes

Name:____________________________________

1. (20 pts) Obtain the eigenvalues for the system given by

$$\begin{aligned} \frac{dT}{dt} + 3T &= \frac{1}{2}C + 1 \\ \frac{dC}{dt} &= -5C + T + 2 \end{aligned}$$

2. (20 pts) Obtain a second order model that matches the data given in Figure 1, where $T(0) = 30^oC$ and $dT/dt(0) = 0^oC/s$.

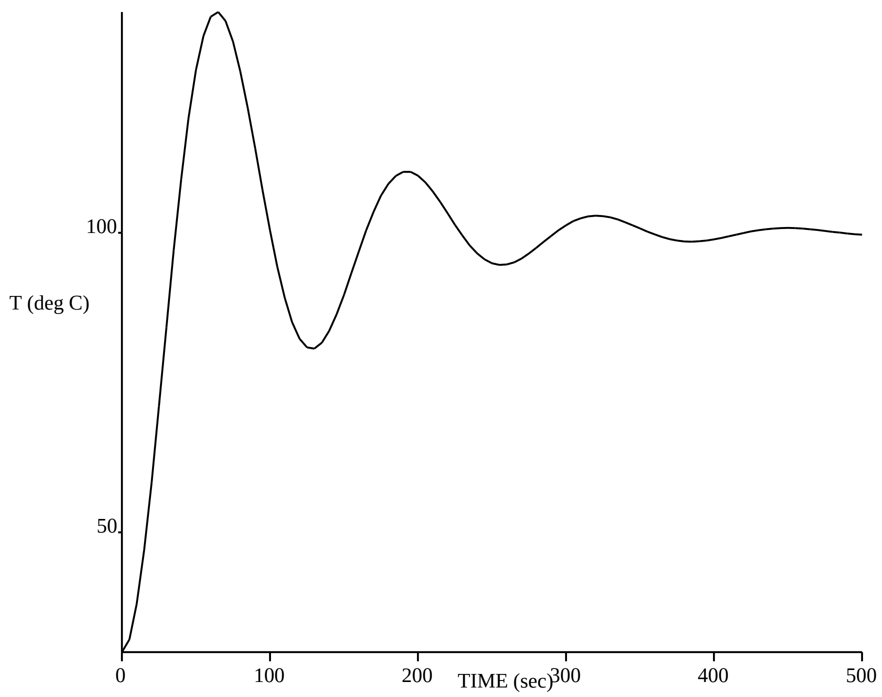

Figure 1: The response of an underdamped second order system.

3. (20 pts) Find the maximum value of k such that the system described by

$$\frac{dx}{dt} + 3 = (2k - 4)x$$

is stable.

4. (20 pts) Linearize the following system:

$$\begin{aligned}\frac{dx}{dt} &= -xy^2 + 2\sqrt{u} \\ \frac{dy}{dt} &= 3x^2 - uy\end{aligned}$$

around the point: $x = 1$, $y = 1$ and $u = 0.5$

5. Two storage tanks are connected in series, in which part of the feed to tank 1 is a recycle stream as shown in Figure 2. The recycle flow rate is a fraction α of the flow out of tank 2. The volumeteric flow rate out of tank 1 is a function of the height in tank 1, $F_1 = k_1\sqrt{h_1}$, where k_1 is a constant. The volumeteric flow rate out of tank 2 is a function of the height in tank 2, $F_2 = k_2\sqrt{h_2}$, where k_2 is a constant. The volumetric flow rate of fresh feed is given by F_o. The cross-sectional area of tank 1 is A while the cross-sectional area of tank 2 is $1.5A$.

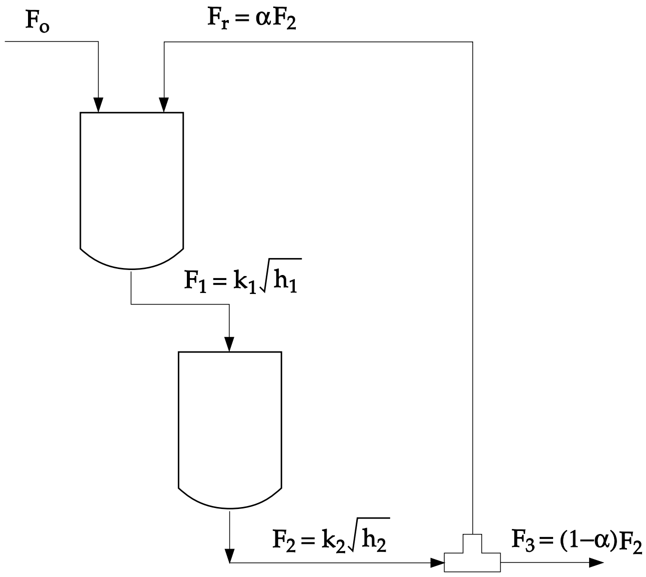

Figure 2: Storage tank with recycle.

(a) (20 pts) Derive a dynamic process model for h_1 and h_2.

(b) (Bonus: 10 pts) Assuming F_o is constant, obtain the steady state values of h_1 and h_2 in terms of the other quantities.