

CM 416 First Exam (Open-book open-notes)
December 16, 1999 7-9pm

Name: _____ **Box No.** _____

1. (20 pts) A cooling process is known to obey a first order process with a time constant of 2 hours. Further, the external effect Q is held constant. The model is given as

$$\frac{dT}{dt} = -\frac{1}{2}T + Q$$

Starting at initial condition of $T(0)=150^{\circ}\text{C}$, the temperature was found to be 80°C after 2 hours has elapsed from the starting time. Estimate the value of Q .

2. For the system described by the following equations

$$\frac{dx}{dt} = -0.3x + y + 0.1u$$

$$\frac{dy}{dt} = 0.1x - 0.1u$$

a proportional controller was implemented with $u = k_c(x_{\text{set}} - x)$.

- a) (15 pts) Show that with the controller in place, the resulting characteristic equation for the system is given by

$$s^2 + (0.3 + 0.1k_c)s - 0.1(1 + k_c) = 0$$

- b) (5 pts) Determine the stability of the system under proportional control, when $k_c = 0$, $k_c = -1.5$ and $k_c = -4.0$.

3. (20 pts) The following equations represent the jacketed CSTR process (Figure 1) linearized around the operating conditions,

$$\frac{dC}{dt} = \alpha (C_{in} - C) - \beta C - \eta T$$

$$\frac{dT}{dt} = \gamma (T_{in} - T) + \mu C - \kappa (T - T_{cool}) + \xi T$$

where C is the concentration in the reactor, T is the temperature of the reactor, T_{in} is the temperature of the feed, C_{in} is the concentration of the feed, T_{cool} is the temperature of the cooling water.

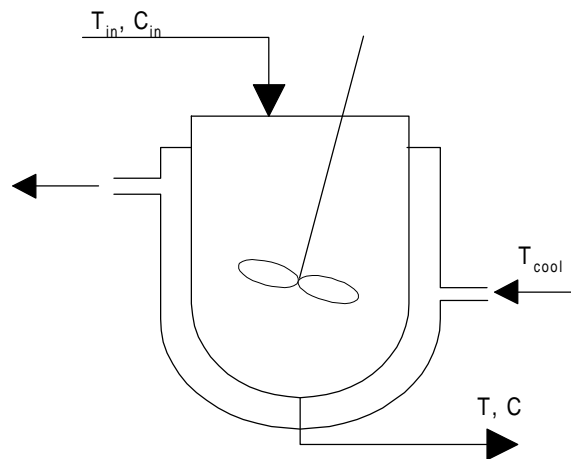


Figure 1. Jacketed CSTR

Suppose the parameters have been found to have the following values:

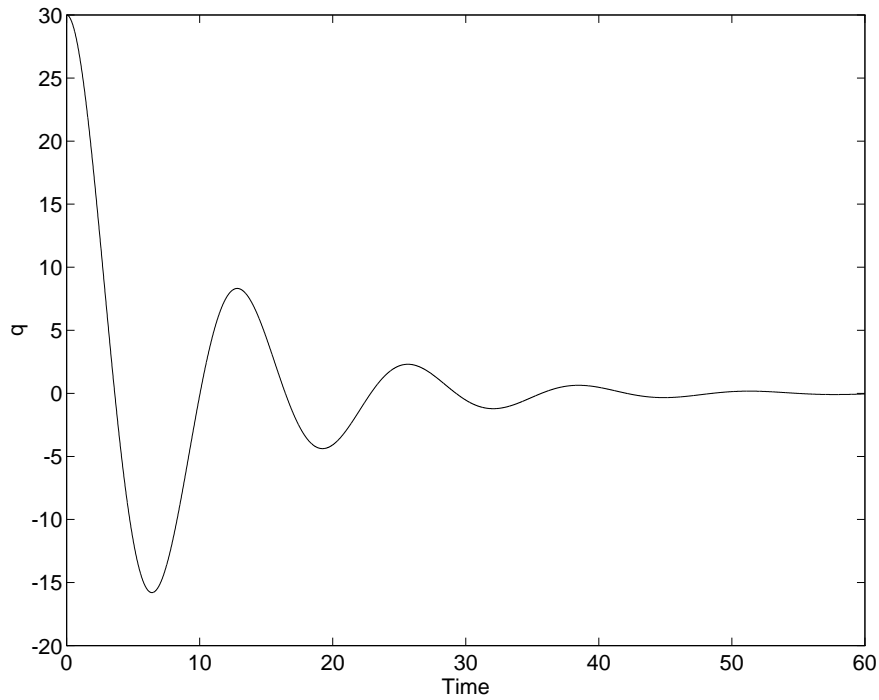
α	1.0
β	0.5
γ	1.0
η	0.1
μ	2.0
ξ	5.0

The parameter κ contains the heat transfer coefficient of the cooling jacket. Determine the minimum value of κ for which the system will be stable.

4. A second order process described by

$$\frac{d}{dt} \left(R \frac{dq}{dt} + \alpha q \right) + \beta q = (R-1)^2$$

has a response shown in Figure 2 in which $q(0)=10$ and $dq/dt(0)=0$.



- a) (20 pts) Determine the parameters R , α and β .
 b) (Bonus:5 pts) Is the system underdamped or overdamped ?
5. (20 pts) Obtain a linearized equation for the process described by

$$\frac{dT}{dt} + T = \frac{1.23 P}{T + 2}$$

around $T=T_0=50$ and $P=P_0=1.5$.