First Exam
CM3310
February 6, 2001

Name: ____ **Box No.**________

1. The height of liquid inside the spherical tank shown in Figure 1 can be modelled to be

$$\frac{dh}{dt} = \frac{F_o - k_v u \sqrt{h}}{\pi\, r^2}$$

where

$$r = \sqrt{2Rh - h^2}$$

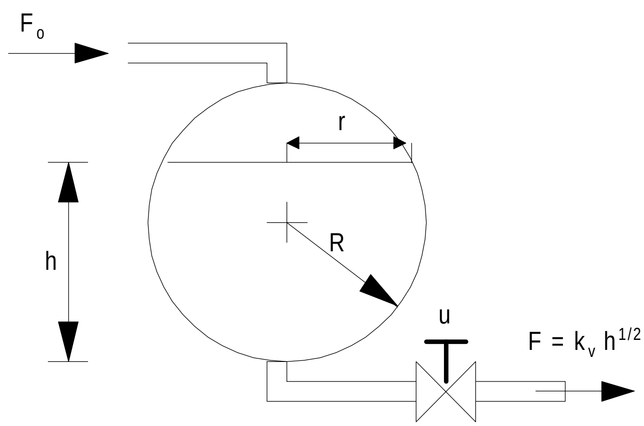

Figure 1

with parameters fixed at R=20 ft and k_v=1.3 (ft^3/min)(ft$^{-1/2}$).

a) (10 pts) Find the steady state height corresponding to $F_{o,ss}$=2 ft^3/min and u_{ss}=0.5.
b) (20 pts) Obtain a linearized model for height, h, around the steady state found in problem (a). (Note: treat F_o as a disturbance variable instead of a constant.)

2. The dynamic model of for the temperature of two tanks are given by

$$10\frac{dT_1}{dt} = (100 - T_1) + 10T_2$$

$$5\frac{dT_2}{dt} = (T_1 - T_2) + 2u$$

Using a proportional control, with u_o=0.5 and $T_{2\ set}$ = 30,

$$u = 0.5 + k_c(30 - T_2)$$

a) (20 pts) Find the range of values for proportional control gain, k_c, that would stabilize the process
b) (20 pts) Using a value for k_c=30, calculate the steady state offset for T_2, with $T_{2\ set}$ = 30.

3. (30 pts) The dynamic model for pressure in tank is given by:

$$\frac{dP}{dt} = -3P + 2u$$

Using a PI control, with u_o=10,

$$u = 10 + k_c \left([P_{set} - P] + \frac{1}{\tau_I} \int [P_{set} - P] dt \right)$$

With k_c=1.0, find the range of values for τ_I that will make the process underdamped.

4. (Bonus:10 pts) The nonlinear process dynamics for z is given by the following:

$$(1+z)\frac{d^3 z}{dt^3} + 3\left(\frac{dz}{dt}\right)^3 = \frac{d^2 z}{dt^2} \qquad (1)$$

By introducing the following variables,

$$y = \frac{dz}{dt} \qquad\qquad x = \frac{d^2 z}{dt^2}$$

and fixing Δt=0.1 sec, determine the functions $f(x_k, y_k, z_k)$, $g(x_k, y_k, z_k)$ and $h(x_k, y_k, z_k)$ for the recursion equations\

$$\begin{aligned} z_{k+1} &= z_k + f(x_k, y_k, z_k) \\ y_{k+1} &= y_k + g(x_k, y_k, z_k) \\ x_{k+1} &= x_k + h(x_k, y_k, z_k) \end{aligned}$$

based on Euler's method for numerical solution of equation (1).