SOLUTION TO EXAM 1, Dec. 16, 1999

1. $$\frac{T(\tau) - T(0)}{T_{ss} - T(0)} = 0.632$$

$$T_{ss} = T(0) + \frac{(T(\tau) - T(0))}{0.632} = 150 + \left(\frac{80 - 150}{0.632}\right) = 39.241$$

$$Q = \frac{T_{ss}}{2} = \frac{39.241}{2} = 19.62$$

2. $$\frac{dx}{dt} + \left(0.3 + 0.1 \cdot k_c\right) \cdot x - y = 0.1 \cdot k_c \cdot x_{set}$$

$$\frac{dy}{dt} - 0.1 \cdot \left(1 + k_c\right) \cdot x = -0.1 \cdot k_c \cdot x_{set}$$

Characteristic Equation: $$\det\left[\begin{bmatrix} s + 0.3 + 0.1 \cdot k_c & -1 \\ -0.1 \cdot \left(1 + k_c\right) & s \end{bmatrix}\right] = 0$$

$$s^2 + \left(0.3 + 0.1 \cdot k_c\right) \cdot s - 0.1 \cdot \left(1 + k_c\right) = 0$$

$k_c = 0$	$s^2 + 0.3 \cdot s - 0.1 = 0$	unstable
$k_c = -1.5$	$s^2 + 0.15 \cdot s + 0.05 = 0$	stable
$k_c = -4$	$s^2 - 0.1 \cdot s + 0.3 = 0$	unstable

3. $$\frac{dC}{dt} = C_{in} - C - 0.5 \cdot C - 0.1 \cdot T$$

$$\frac{dT}{dt} = \left(T_{in} - T\right) + 2 \cdot C - \kappa \cdot T + \kappa \cdot T_{cool} + 5 \cdot T$$

Rearranging,

$$\frac{dC}{dt} + (1.5)\cdot C + 0.1\cdot T = C_{in}$$

$$\frac{dT}{dt} - 2\cdot C + (\kappa - 4)\cdot T = T_{in} + \kappa\cdot T_{cool}$$

Characteristic Equation: $$\det\left(\begin{bmatrix} s+1.5 & 0.1 \\ -2 & s+\kappa-4 \end{bmatrix}\right) = 0$$

$$s^2 + (\kappa - 2.5)\cdot s + (1.5\cdot\kappa - 5.8) = 0$$

For stability, $\kappa > 2.5$ and $\kappa > \frac{5.8}{1.5} = 3.867$

Thus $\kappa > 3.867$

4. $$\text{Overshoot} = \frac{2.6}{4.95} = \exp\left(\frac{-\zeta\cdot\pi}{\sqrt{1-\zeta^2}}\right) \qquad \zeta = \sqrt{\frac{\ln\left(\frac{2.6}{4.95}\right)^2}{\pi^2 + \left(\ln\left(\frac{2.6}{4.95}\right)\right)^2}} = 0.201$$

$$\text{Period} = \left(\frac{2.25}{1.75}\cdot 10 = 12.86\right) = \frac{2\cdot\pi\cdot\tau_n}{\sqrt{1-\zeta^2}} \qquad \tau_n = \left(\frac{12.86}{2\cdot\pi}\cdot\sqrt{1-0.201^2} = 2.0\right)$$

Standard Form for Given Process:

$$\left(\frac{R}{\beta}\cdot\frac{d^2}{dt^2}q + \frac{\alpha}{\beta}\cdot\frac{dq}{dt} + q\right) = \frac{(R-1)^2}{\beta}$$

$$\frac{(R-1)^2}{\beta} = 0 \qquad R = 1$$

$$\frac{R}{\beta} = 2^2 \qquad \beta = \frac{1}{4}$$

$$\frac{\alpha}{\beta} = 2\cdot 2.\cdot 0.201 \qquad \alpha = 0.1$$

From the figure, the system is underdamped.

5. $$\frac{dT}{dt} = A + B\cdot(T - 50) + C\cdot(P - 1.5)$$

$$A := \frac{1.23\cdot 1.5}{50 + 2} - 50 \qquad A = -49.965$$

$$B := -1.23\cdot\frac{1.5}{(50 + 2)^2} - 1 \qquad B = -1.001$$

$$C := \frac{1.23}{50 + 2} \qquad C = 0.024$$

$$\frac{dT}{dt} = -49.97 - 1.001\cdot(T - 50) + 0.024\cdot(P - 1.5)$$