Solution to exam 1 v2.0 February 27, 2001

1. $\tau = \frac{1}{5}$

2. $4\cdot\frac{d^2}{dt^2}T + 8\cdot\frac{dT}{dt} + 2\cdot T = 5$

   in standard form, $2\cdot\frac{d^2}{dt^2}T + 4\cdot\frac{dT}{dt} + T = \frac{5}{2}$

   $\tau_n := \sqrt{2}$

   $\zeta := \frac{4}{2\cdot\tau_n}$ $\qquad \zeta = 1.414$ $\qquad$ thus overdamped

3. $\frac{dx}{dt} = (\alpha + 1)\cdot x + 2\cdot y$

   $\frac{dy}{dt} = -3\cdot x - \alpha\cdot y$

$$\frac{d^2}{dt^2}x = (\alpha+1)\cdot\frac{dx}{dt} + 2\cdot\frac{dy}{dt}$$

$$\frac{d^2}{dt^2}x = (\alpha+1)\cdot\frac{dx}{dt} + 2\cdot(-3\cdot x - \alpha\cdot y)$$

$$\frac{d^2}{dt^2}x = (\alpha+1)\cdot\frac{dx}{dt} + 2\cdot\left[-3\cdot x - \alpha\cdot\left[\frac{1}{2}\cdot\left[\frac{dx}{dt} - (\alpha+1)\cdot x\right]\right]\right]$$

$$\frac{d^2}{dt^2}x - \frac{dx}{dt} + (6 - \alpha\cdot(\alpha+1))\cdot x = 0$$

Thus no values of α can stabilize the process.

4. For the required 1/4 decay ratio,

$$\frac{1}{4} = \exp\left(\frac{-2\cdot\pi\cdot\zeta}{\sqrt{1-\zeta^2}}\right) \qquad \zeta := \frac{\ln(2)}{\sqrt{\ln(2)^2 + \pi^2}} \qquad \zeta = 0.215$$

System: $2\cdot\frac{d^2}{dt^2}C + \frac{dC}{dt} = \frac{1}{3}\cdot(2\cdot u - C)$

$$2\cdot\frac{d^2}{dt^2}C + \frac{dC}{dt} = \frac{1}{3}\cdot\left[2\cdot k_c\cdot\left(C_{set} - C\right) - C\right]$$

$$2\cdot\frac{d^2}{dt^2}C + \frac{dC}{dt} + \left(\frac{2\cdot k_c + 1}{3}\right)\cdot C = \frac{2}{3}\cdot k_c\cdot C_{set}$$

$$\left(\frac{6}{2\cdot k_c + 1}\right)\cdot\frac{d^2}{dt^2}C + \left(\frac{3}{2\cdot k_c + 1}\right)\cdot\frac{dC}{dt} + C = \left(\frac{2\cdot k_c}{2\cdot k_c + 1}\right)\cdot C_{set}$$

$$\tau_n = \sqrt{\frac{6}{2\cdot k_c + 1}}$$

$$\zeta = \left(\frac{3}{2\cdot k_c + 1}\right)\cdot\frac{1}{2}\cdot\sqrt{\frac{2\cdot k_c + 1}{6}}$$

$$k_c := \frac{-1}{16}\cdot\frac{\left(8\cdot\zeta^2 - 3\right)}{\zeta^2} \qquad k_c = 3.539$$

5. $$\frac{dC}{dt} = \frac{1}{\tau}\cdot\left(C_{in} - C\right) - k_r\cdot C^2$$

After linearization,

$$\frac{dC}{dt} = \left[\frac{1}{\tau}\cdot\left(C_{inss} - C_{ss}\right) - k_r\cdot C_{ss}^{\;2}\right] - \left(\frac{1}{\tau} + 2\cdot k_r\cdot C_{ss}\right)\cdot\left(C - C_{ss}\right) + \frac{1}{\tau}\cdot\left(C_{in} - C_{inss}\right)$$

$$\frac{dC}{dt} = \left(k_r\cdot C_{ss}^{\;2}\right) - \left(\frac{1}{\tau} + 2\cdot k_r\cdot C_{ss}\right)\cdot C + \frac{1}{\tau}\cdot C_{in}$$

Now compare with given linearized equation:

$$\frac{dC}{dt} = 0.048 - 0.34\cdot C + 0.1\cdot C_{in}$$

$$\frac{1}{\tau} = 0.1 \qquad \tau := 10$$

$$\frac{1}{\tau} + 2\cdot k_r\cdot C_{ss} = 0.34 \qquad k_r = \frac{.12}{C_{ss}}$$

$$k_r\cdot C_{ss}^{\;2} = 0.048$$

$$.12\cdot C_{ss} = 0.048$$

$$C_{ss} := 0.4 \qquad k_r := \frac{0.12}{0.4} \qquad k_r = 0.3$$

$$\frac{1}{\tau}\cdot\left(C_{inss} - C_{ss}\right) - k_r\cdot C_{ss}^{\;2} = 0$$

$$C_{inss} := -\left(\frac{-1}{\tau}\cdot C_{ss} - k_r\cdot C_{ss}^{\;2}\right)\cdot\tau \qquad C_{inss} = 0.88$$

6. $$\frac{dx}{dt} + 2\cdot x = 0.5\cdot u \qquad u = k_c\cdot\left[\left(x_{set} - x\right) - \tau_D\cdot\frac{de}{dt}\right] = 2\cdot\left(1 - x - 5\cdot\frac{dx}{dt}\right)$$

$$\frac{dx}{dt} + 2\cdot x = 1 - x - 5\cdot\frac{dx}{dt}$$

$$2\cdot x_{ss} = 1 - x_{ss}$$

$$x_{ss} = \frac{1}{3} \qquad \text{Offset} = 1 - \frac{1}{3} \neq 0$$