

CM 3310
Second Exam
March 20, 2001

Name: _____ Box No. _____

1. Three continuously stirred tank reactors are connected in series as shown in Figure 1. The model for the process is described by the following equations:

$$\frac{dC_{A1}}{dt} = \frac{1}{\tau_1} (C_{Ain} - C_{A1}) - k_1 C_{A1}$$

$$\frac{dC_{A2}}{dt} = \frac{1}{\tau_2} (C_{A1} - C_{A2}) - k_2 C_{A2}$$

$$\frac{dC_{A3}}{dt} = \frac{1}{\tau_3} (C_{A2} - C_{A3}) - k_3 C_{A3}$$

where the feed concentration C_{Ain} is manipulated by using feedback information of C_{A3} from tank 3.

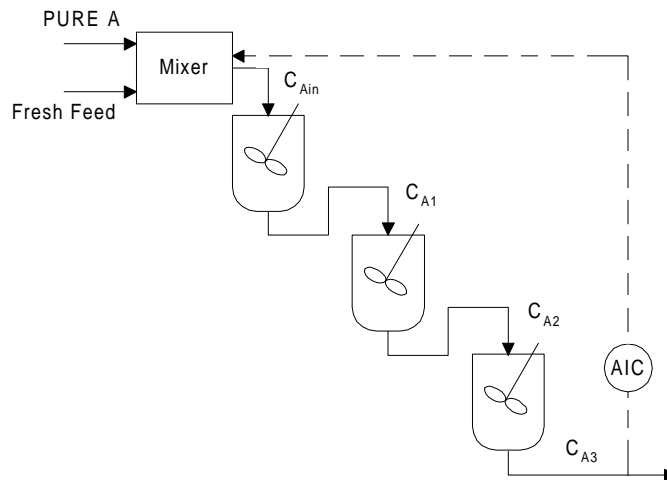


Figure 1. Three CSTRs in series.

Assuming that the mixer controller is ideal, we could then simplify the feedback system to one that is described by the block diagram shown in Figure 2.

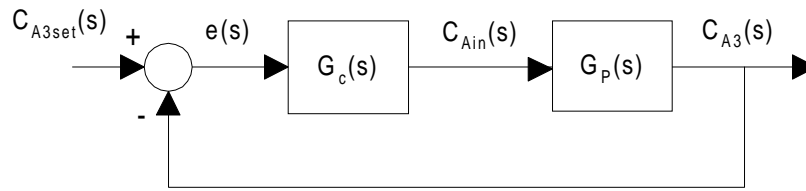


Figure 2. Feedback controlled system.

- a) (25 pts) Find the transfer function, $G_p(s)$, from C_{Ain} to C_{A3} in terms of the parameters, $\tau_1, \tau_2, \tau_3, k_1, k_2$ and k_3 . (Assume that all the variables are perturbation variables and that the initial conditions are all zero.)
- b) (25 pts) Letting $\tau_1=\tau_2=\tau_3=10.0$ and $k_1=k_2=k_3=0.2$, the open loop transfer function is given by

$$G_p(s) = \frac{1}{(10s + 3)^3}$$

Using a proportional controller, find the ultimate gain for the process.

2. In some processes that contain large deadtimes, the controllers will be limited to low gain controllers. To compensate for this, a Smith Predictor is usually included in the control system such as the one shown in Figure 4 to improve the performance.

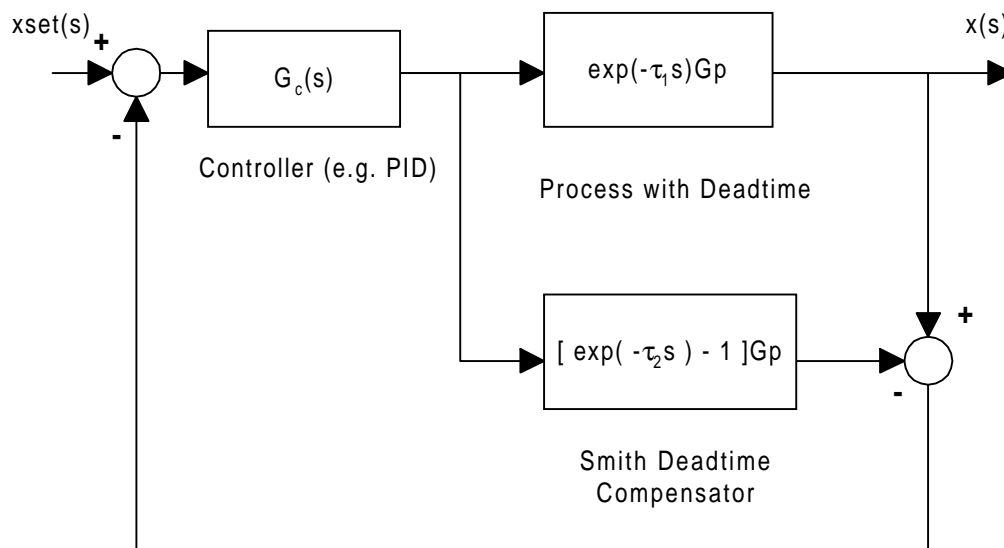


Figure 4. Feedback Process with a Smith Predictor.

- a) (25 pts) Show that if the values of the time delays in the process and the predictor are exactly equal, i.e. $\tau_1 = \tau_2$, then the closed loop transfer function from $x_{set}(s)$ to $x(s)$ is given by

$$G_{CL,ideal}(s) = e^{-\tau_1 s} \frac{G_c G_p}{1 + G_c G_p}$$

Thus allowing higher gains to be implemented for better performance.

- b) (25 pts) Unfortunately one can not model the dead time exactly. It is recommended that one should in fact overestimate the dead time a little, i.e. $\tau_2 > \tau_1$. One would then still need to check for stability.

Let $G_p = 2/(50s+1)$. The closed loop transfer function if $\tau_2 > \tau_1$ is given by

$$G_{CL,nonideal}(s) = e^{-\tau_1 s} \frac{2G_c}{50s + 1 + 2G_c(1 + e^{-\tau_1 s} - e^{-\tau_2 s})}$$

Since the denominator is no longer a simple polynomial, one may have to resort to an approximation. One such approximation for an exponential function is the Padé approximation given by,

$$\exp(-\tau_d s) \cong \frac{-\frac{\tau_d}{2}s + 1}{\frac{\tau_d}{2}s + 1}$$

Using the Pade Approximation and a proportional controller for $G_c = K_c$, for $\tau_1 = 20$ and $\tau_2 = 22$, the closed loop transfer function reduces to

$$G_{cl} = \frac{(-10 \cdot s + 1) \cdot [2 \cdot (11 \cdot s + 1) \cdot K_c]}{(5500 \cdot s^3 + 1160 \cdot s^2 + 71 \cdot s + 1 + 220 \cdot K_c \cdot s^2 + 46 \cdot K_c \cdot s + 2 \cdot K_c)}$$

Determine the range of values of K_c that would stabilize the process.

3. (Bonus: 10pts) Separate the following transfer function into partial fractions

$$G(s) = \frac{2s + 1}{(s + 1)([s + 2]^2 + 4)}$$