CM 3310
Exam 2
April 1, 2003
7:30-9:30pm
Open book Open Notes

Name: _____ Box No. _____

1. a) (30 pts) Obtain the equivalent transfer function, G_{cl}, from $\hat{T}_{set}(s)$ to $\hat{T}(s)$ (in terms of transfer functions A, B, C and D) for the block diagram shown in Figure 1.

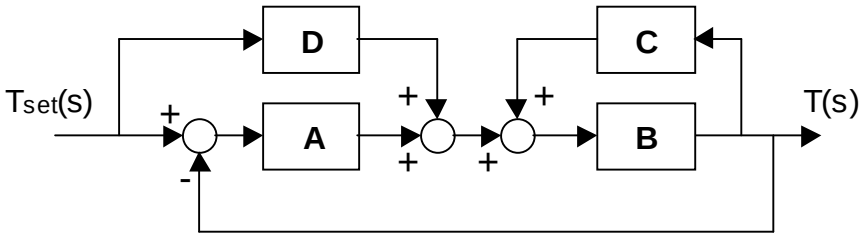

Figure 1.

b) (30 pts) Suppose the transfer functions in Figure 1 are given by:

$$A = \frac{K}{5s+1} \qquad B = \frac{1}{-3s+1} \qquad C = \frac{2}{s+1} \qquad D = \frac{1-A}{B}\left(\frac{1}{s+1}\right)$$

then the equivalent transfer function, G_{cl}, from $\hat{T}_{set}(s)$ to $\hat{T}(s)$ will be

$$\hat{T}(s) = G_{cl}(s)\,\hat{T}_{set}(s)$$

$$G_{cl} = \frac{15s^2 + (-2-4K)s - 1}{15s^3 + 13s^2 + (7-K)s + 1 - K}$$

Determine the range of K that would maintain the stability of the transfer function from $\hat{T}_{set}(s)$ to $\hat{T}(s)$.

2. (30 pts) Two continuously stirred tank reactors are connected in series as shown in Figure 2.

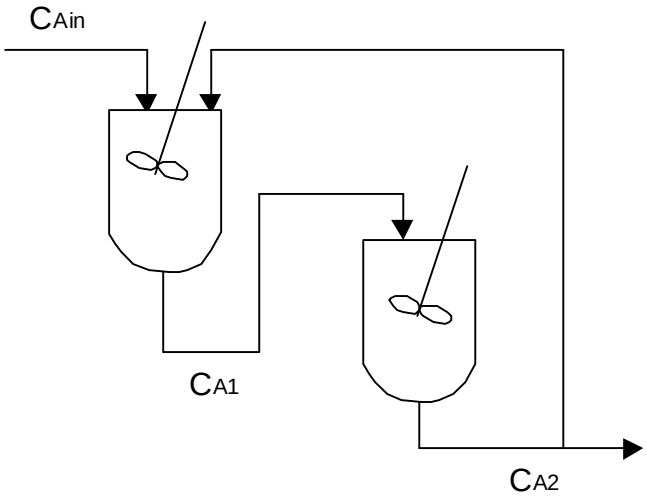

Figure 2.

The model for the process is described by the following equations:

$$\frac{dC_{A1}}{dt} = \frac{1}{\tau_1}\left(C_{Ain} - C_{A1} + \alpha C_{A2}\right) - k_1 C_{A1}$$

$$\frac{dC_{A2}}{dt} = \frac{1}{\tau_2}\left(C_{A1} - C_{A2}\right) - k_2 C_{A2}$$

where the feed concentration is given by C_{Ain}.

a) (25 pts) Assuming zero initial conditions, show that the transfer function from $C_{Ain}(s)$ to $C_{A2}(s)$, with the following parameter values,

τ_1	1
τ_2	2
k_1	0.2
k_2	0.3
α	0.1

is given by

$$\hat{C}_{A2}(s) = \left[\frac{1}{2s^2 + 4s + 1.82}\right]\hat{C}_{Ain}(s)$$

b) (15 pts) Suppose C_{Ain} (t) is given by

$$C_{Ain}(t) = 0.4 + 0.5e^{-t}$$

Solve for C_{A2} as a function of time.

3. (Bonus: 5 pts) Determine the Laplace transform of

$$f(t) = \left(5 - 2\sin\left(\frac{2\pi t}{10}\right) \right) e^{-2t}$$