Solution to Exam 2 Jan 30, 1995

1. $T(s)=\left(\frac{1}{s+3}\right)\cdot q(s)+\left(\frac{\alpha\cdot 4}{s+3}\right)\cdot T_r(s)$

$T_r(s)=\left(\frac{2}{s+2}\right)\cdot T(s)$

$$T(s)=\left[\frac{\frac{1}{s+3}}{1-\left(\frac{\alpha\cdot 4}{s+3}\right)\cdot\left(\frac{2}{s+2}\right)}\right]\cdot q(s)=\left[\frac{(s+2)}{\left(s^2+5\cdot s+6-8\cdot\alpha\right)}\right]\cdot q(s)$$

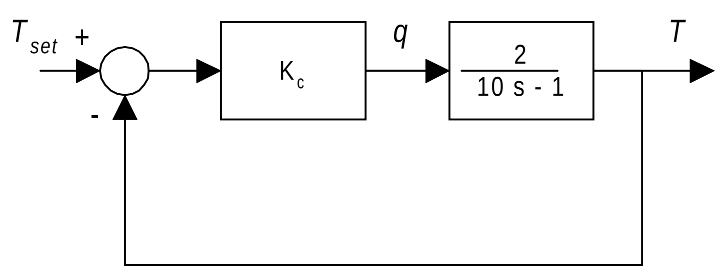
$$G_{cl}=\frac{(s+2)}{\left[s^2+5\cdot s+(6-8\cdot\alpha)\right]}$$

For stability, $\alpha<\frac{3}{4}$

2. a) $T(s)=\left(\frac{2}{10\cdot s-1}\right)\cdot q(s)$

eigenvalue: $s:=0.1$ thus is unstable

b)



c) $T(s)=\left(\frac{2\cdot K_c}{10\cdot s-1+2\cdot K_c}\right)\cdot T_{set}$

$K_c>0.5$ for stability

3. a) $G_P(s) = \frac{-s+1}{s-3}$

$$y(s) = \left(\frac{K_{inner} \cdot G_P}{1 + K_{inner} \cdot G_P}\right) \cdot x(s) = \left[\frac{K_{inner} \cdot (1-s)}{\left(1 - K_{inner}\right) \cdot s - 3 + K_{inner}}\right] \cdot x(s)$$

$$G_{CL,INNER} = \frac{K_{inner} \cdot (1-s)}{\left(1 - K_{inner}\right) \cdot s - 3 + K_{inner}}$$

root: $s = \frac{\left(3 - K_{inner}\right)}{\left(1 - K_{inner}\right)}$ for s<0, we need $1 < K_{inner} < 3$

b) $y(s) = \left(\frac{G_c \cdot G_{CL,INNER}}{1 + G_c \cdot G_{CL,INNER}}\right) \cdot y_{set}(s)$

For PI control, $G_c = \frac{K_c \cdot \left(t_I \cdot s + 1\right)}{t_I \cdot s} = \frac{-1}{40} \cdot \frac{(10 \cdot s + 1)}{s}$

with K_{inner} = 2 $G_{CL,INNER} = \frac{(-2 \cdot s + 2)}{(-s - 1)}$

$$y(s) = \left[\frac{\left(-10 \cdot s^2 + 9 \cdot s + 1\right)}{\left(10 \cdot s^2 + 29 \cdot s + 1\right)}\right] \cdot y_{set}(s)$$

eigenvalues: $\begin{pmatrix} -0.035 \\ -2.865 \end{pmatrix}$ thus the system is stable.

4. $\left(s^2 \cdot x(s) - s \cdot x_o - dx_o\right) + \left(s \cdot x(s) - x_o\right) + x(s) = \left(\frac{1}{s+1}\right)$

$$\left(s^2 + s + 1\right) \cdot x(s) = \frac{1}{s+1} + (s+1) \cdot x_o + dx_o$$

$$x(s) = \frac{1 + (s+1)^2 \cdot x_o + (s+1) \cdot dx_o}{(s+1) \cdot \left(s^2 + s + 1\right)}$$

$$.= \frac{A}{s+1} + B\cdot\frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} + C\cdot\frac{\frac{\sqrt{3}}{2}}{\left(s+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$A = \lim_{s \to -1} \frac{1 + (s+1)^2\cdot x_o + (s+1)\cdot dx_o}{\left(s^2+s+1\right)}$$

$$.= 1$$

$$Q = \lim_{s \to -\frac{1}{2} + i\cdot\frac{\sqrt{3}}{2}} \frac{1 + (s+1)^2\cdot x_o + (s+1)\cdot dx_o}{(s+1)}$$

$$.= \left(\frac{1}{2} + dx_o + \frac{1}{2}\cdot x_o\right) + \left(\frac{\sqrt{3}}{2}\cdot x_o - \frac{\sqrt{3}}{2}\right)\cdot i$$

$$B = \frac{2}{\sqrt{3}}\cdot\left(\frac{\sqrt{3}}{2}\cdot x_o - \frac{\sqrt{3}}{2}\right) = x_o - 1$$

$$C = \frac{2}{\sqrt{3}}\cdot\left(\frac{1}{2} + dx_o + \frac{1}{2}\cdot x_o\right) = \frac{1}{3}\cdot\sqrt{3}\cdot\left(1 + 2\cdot dx_o + x_o\right)$$

$$x(t) = e^{-t} + e^{-\frac{t}{2}}\cdot\left[\left(x_o - 1\right)\cdot\cos\left(\frac{\sqrt{3}}{2}\cdot t\right) + \frac{\sqrt{3}}{3}\cdot\left(1 + 2\cdot dx_o + x_o\right)\cdot\sin\left(\frac{\sqrt{3}}{2}\cdot t\right)\right]$$

5) $\sin(\omega\cdot t)\cdot t\cdot\exp(-a\cdot t) = \frac{\exp(i\cdot\omega\cdot t) - \exp(-i\cdot\omega\cdot t)}{2i}\cdot t\cdot\exp(-a\cdot t)$

$$.= \frac{1}{2i}\cdot t\cdot\exp((-a + i\cdot\omega)\cdot t) - \frac{1}{2i}\cdot t\cdot\exp((-a - i\cdot\omega)\cdot t)$$

Using the first shifting theorem,

$$\text{Laplace}(\sin(\omega\cdot t)\cdot t\cdot \exp(-a\cdot t)) = \frac{1}{2i}\cdot\left[\frac{1}{(s+a-i\cdot\omega)^2}\right] - \frac{1}{2i}\cdot\left[\frac{1}{(s+a+i\cdot\omega)^2}\right]$$

$$= \frac{1}{2i}\cdot\left[\frac{(s+a+i\cdot\omega)^2 - (s+a-i\cdot\omega)^2}{\left[(s+a)^2+\omega^2\right]^2}\right]$$

$$= \frac{2\cdot\omega\cdot(s+a)}{\left[(s+a)^2+\omega^2\right]^2}$$