Solution to Exam 2 Jan 27 1997

1) $t_0 := 2 \quad t_2 := 4 \quad t_3 := 4.6$

$$4 - t_1 = \ln(2)\cdot(4.6 - t_1)$$

$$t_1 := 2.64$$

$$K := -20$$

$$\tau := 4.6 - 2.64 \qquad \tau = 1.96$$

$$\tau_{del} := 2.64 - 2 \qquad \tau_{del} = 0.64$$

$$K_c := \frac{1}{K}\cdot\frac{\tau}{\tau_{del}}\cdot\left(0.9 + \frac{1}{12}\cdot\frac{\tau_{del}}{\tau}\right) \qquad K_c = -0.142$$

$$t_I := \tau_{del}\cdot\left[\frac{30 + 3\cdot\frac{\tau_{del}}{\tau}}{9 + 20\cdot\frac{\tau_{del}}{\tau}}\right] \qquad t_I = 1.277$$

2. a) $G_{cl} = \frac{Kc\cdot G_P}{1 + K_c\cdot G_P} = (-s + a)\cdot\frac{Kc}{\left[s^2 + (-K_c + b + c)\cdot s + K_c\cdot a + b\cdot c\right]}$

Characteristic Equation: $s^2 + (-K_c + b + c)\cdot s + (K_c\cdot a + b\cdot c) = 0$

b) Routh-Hurwitz Array:

$$\begin{bmatrix} 1 & (K_c\cdot a + b\cdot c) \\ -K_c + b + c & 0 \\ (K_c\cdot a + b\cdot c) & 0 \end{bmatrix}$$

@ Kc=b+c,

$$\begin{pmatrix} 1 & a\cdot b + a\cdot c + b\cdot c \\ 0 & 0 \\ a\cdot b + a\cdot c + b\cdot c & 0 \end{pmatrix}$$

the roots are,

$i\cdot\sqrt{a\cdot b + a\cdot c + b\cdot c}$ and $-i\cdot\sqrt{a\cdot b + a\cdot c + b\cdot c}$

Thus, $K_u = b + c$

$$P_u = \frac{2\cdot\pi}{\sqrt{a\cdot b + a\cdot c + b\cdot c}}$$

Based on Ziegler-Nichols method,

$$K_c = \frac{b + c}{2.2}$$

$$t_I = \frac{2\cdot\pi}{1.2\cdot\sqrt{a\cdot b + a\cdot c + b\cdot c}}$$

3. $$y(s) = \frac{1}{5}\cdot x_o(s) + \frac{2}{5}\cdot x_1(s)$$

$$\left(s + \frac{3}{5}\right)\cdot x_c(s) + \frac{4}{5}\cdot x_1(s) = u(s)$$

$$s\cdot x_1(s) = x_o(s)$$

$$\frac{u}{\left(s^2 + \frac{3}{5}\cdot s + \frac{4}{5}\right)}$$

Combining,

$$\left(s + \frac{3}{5}\right)\cdot\left(s\cdot x_1(s)\right) + \frac{4}{5}\cdot x_1(s) = u(s)$$

$$x_1(s) = \left(\frac{1}{s^2 + \frac{3}{5}\cdot s + \frac{4}{5}}\right)\cdot u(s)$$

$$x_c(s) = s\cdot x_1(s) = \left(\frac{s}{s^2 + \frac{3}{5}\cdot s + \frac{4}{5}}\right)\cdot u(s)$$

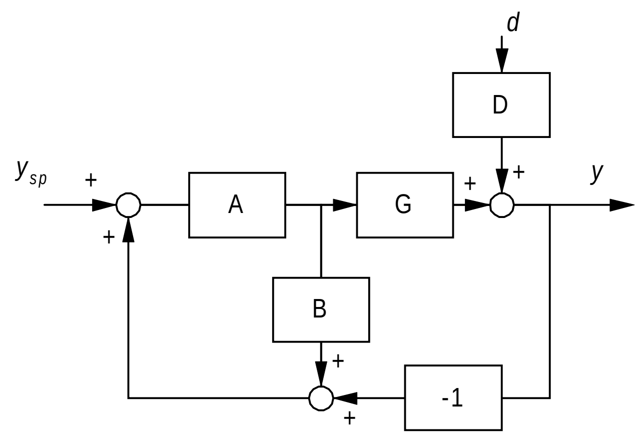
$$u\cdot\frac{(s + 2)}{\left(5\cdot s^2 + 3\cdot s + 4\right)}$$

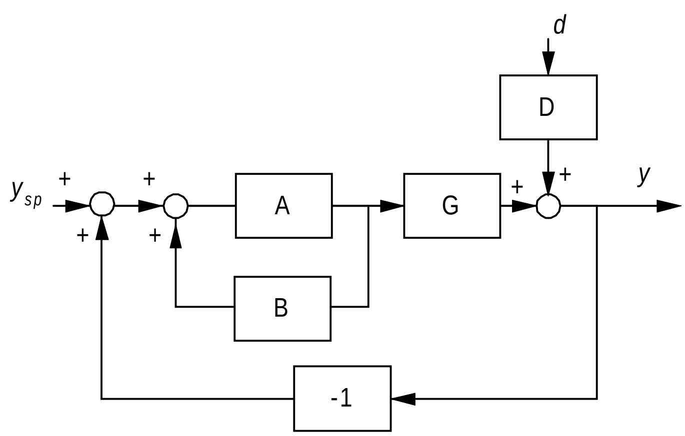
substituting to y(s),

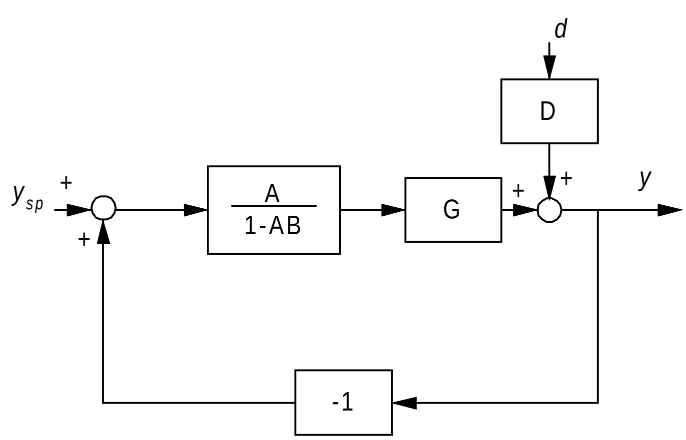
$$y(s) = \frac{1}{5}\cdot\left[\left(\frac{s}{s^2 + \frac{3}{5}\cdot s + \frac{4}{5}}\right)\cdot u(s)\right] + \frac{2}{5}\cdot\left[\left(\frac{1}{s^2 + \frac{3}{5}\cdot s + \frac{4}{5}}\right)\cdot u(s)\right] = \left(\frac{s + 2}{5\cdot s^2 + 3\cdot s + 4}\right)\cdot u(s)$$

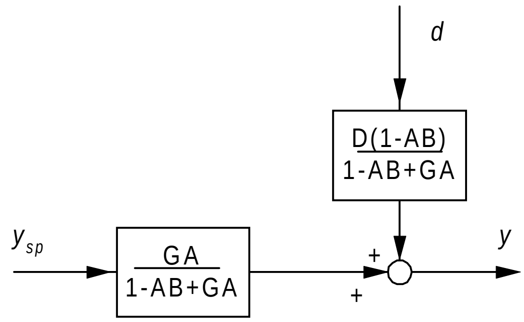
Thus the transfer function from u(s) to y(s) is: $G = \dfrac{s + 2}{5\cdot s^2 + 3\cdot s + 4}$

4.









so the transfer function from ysp to y is : $$G_1 = \frac{G \cdot A}{1 - A \cdot B + G \cdot A}$$

the transfer function from d to y is: $$G_2 = \frac{D \cdot (1 - A \cdot B)}{1 - A \cdot B + G \cdot A}$$

5. $$s^4 + 2 \cdot s^3 + (6 - k) \cdot s^2 + (k - 2) = 0$$

$$\begin{bmatrix} 1 & 6-k & k-2 \\ 2 & 0 & 0 \\ 6-k & k-2 & 0 \\ 2 \cdot \frac{2-k}{6-k} & 0 & 0 \\ k-2 & 0 & 0 \end{bmatrix}$$

There are no value of k that would avoid sign changes, i.e. no k will make the roots have all negative real parts.

6. First note that $$\alpha^{\beta \cdot t} = \exp(\ln(\alpha) \cdot \beta \cdot t)$$

thus, $$f(t) = k \cdot \exp(-(\ln(\alpha) + 1) \cdot \beta \cdot t)$$

$$L(f) = \frac{k}{s + \beta \cdot (\ln(\alpha) + 1)}$$