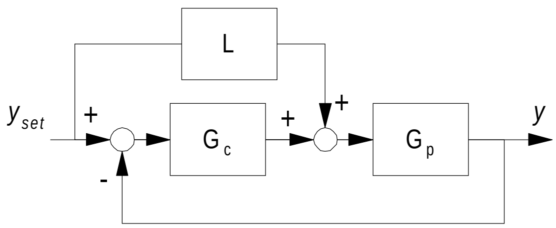
Solution to Exam 2:

1.



$$y = Gp\cdot(L\cdot yset + Gc\cdot(yset - y))$$

$$y = \frac{-(-Gp\cdot L\cdot yset - Gp\cdot Gc\cdot yset)}{(1 + Gp\cdot Gc)}$$

$$y = \frac{(Gp\cdot L + Gp\cdot Gc)}{(1 + Gp\cdot Gc)}\cdot yset$$

with, $L = \frac{1}{3}$ $\quad G_c = K_c$ $\quad G_p = \frac{3}{(20\cdot s + 1)\cdot(s + 1)\cdot(5\cdot s + 1)}$

$$L(y) = \frac{\left(1 + 3\cdot K_c\right)}{\left(100\cdot s^3 + 125\cdot s^2 + 26\cdot s + 1 + 3\cdot K_c\right)}\cdot L(yset)$$

Using Routh-Hurwitz:

$$\begin{bmatrix} 100 & 26 \\ 125 & 1 + 3\cdot K_c \\ 26 - \frac{100}{125}\cdot\left(1 + 3\cdot K_c\right) & 0 \\ 1 + 3\cdot K_c & 0 \end{bmatrix}$$

$-\frac{1}{3} < K_c < \frac{21}{2}$ for stability

using the final value theorem, for yset=$\alpha*\mu(t)$

$$y(\infty) = \lim_{s \to 0} s\cdot\left[\frac{\left(1 + 3\cdot K_c\right)}{\left(100\cdot s^3 + 125\cdot s^2 + 26\cdot s + 1 + 3\cdot K_c\right)}\right]\cdot\frac{\alpha}{s} = \alpha$$

$$\text{offset}(\infty) = \alpha - \alpha = 0$$

2. Take Laplace transform of both sides,

$$s \cdot L(x_1) = -3 \cdot L(x_1) + L(x_2) + 2 \cdot L(u)$$

$$s \cdot L(x_2) = 2 \cdot L(x_1) - 2 \cdot L(x_2) + L(u)$$

rearranging,

$$(s+3) \cdot L(x_1) - L(x_2) = 2 \cdot L(u) \qquad \text{(eq 1)}$$

$$-2 \cdot L(x_1) + (s+2) \cdot L(x_2) = L(u) \qquad \text{(eq 2)}$$

2(eq 1)+ (s+3)(eq 2) will reduce to:

$$((s+2) \cdot (s+3) - 2) \cdot L(x_2) = (s+7) \cdot L(u)$$

$$L(x_2) = \frac{s+7}{(s+2) \cdot (s+3) - 2} \cdot L(u)$$

(s+2)(eq 1) + (eq 2) will reduce to:

$$((s+2) \cdot (s+3) - 2) \cdot L(x_1) = (2 \cdot s + 5) \cdot L(u)$$

$$L(x_1) = \frac{2 \cdot s + 5}{(s+2) \cdot (s+3) - 2} \cdot L(u)$$

Finally, $L(y) = L(x_1) + L(x_2) = \dfrac{3 \cdot s + 12}{(s+2) \cdot (s+3) - 2} \cdot L(u)$

$$L(y) = \frac{3 \cdot s + 12}{s^2 + 5 \cdot s + 4} \cdot L(u)$$

3. $K_u := 6.4$ $\qquad P_u := 9.1$

$K := \dfrac{K_u}{\ldots}$ $\qquad K = 3.765$

$K_c := \frac{\quad}{1.7}$ $\qquad K_c = 3.765$

$t_I := \frac{P_u}{2}$ $\qquad t_I = 4.55$

$t_D := \frac{P_u}{8}$ $\qquad t_D = 1.137$

4. $$\frac{5\cdot s - 1}{(s+1)\cdot\left[(s+2)^2 + 4^2\right]\cdot s} = \frac{A}{s} + \frac{B}{s+1} + C\cdot\frac{s+2}{\left[(s+2)^2+4^2\right]} + D\cdot\frac{4}{(s+2)^2+4^2}$$

$$A = \lim_{s \to 0} \frac{5\cdot s - 1}{(s+1)\cdot\left[(s+2)^2+4^2\right]} = \frac{-1}{20} = -0.05$$

$$B = \lim_{s \to -1} \frac{5\cdot s - 1}{\left[(s+2)^2+4^2\right]\cdot s} = \frac{6}{17} = 0.353$$

$$Q = \lim_{s \to -2+4i} \frac{5\cdot s - 1}{(s+1)\cdot s} = \frac{-43}{170} - \frac{103}{85}\cdot i$$

$$C = \frac{1}{4}\cdot \mathrm{Im}(Q) = -\frac{103}{340} = -0.303$$

$$D = \frac{1}{4}\cdot \mathrm{Re}(Q) = \frac{-43}{680} = -0.063$$

thus,

$$x(t) = -0.05\cdot\mu(t) + 0.353\cdot\exp(-t) + \exp(-2\cdot t)\cdot(-0.303\cdot\cos(4\cdot t) - 0.063\cdot\sin(4\cdot t))$$

5. $f(t) = 3\cdot\sin\left(2\cdot t + \frac{\pi}{2}\right) = 3\cdot\cos(2\cdot t)$ $\qquad L(f) = 3\cdot\frac{s}{s^2+4}$