Solution to Exam 2, March 20, 2001

1. a) $$G_p(s) = \frac{1}{(\tau_1 \cdot s + 1 + \tau_1 \cdot k_1) \cdot (\tau_2 \cdot s + 1 + \tau_2 \cdot k_2) \cdot (\tau_3 \cdot s + 1 + \tau_3 \cdot k_3)}$$

   b) $$G_{cl}(s) = \frac{K_c}{(10 \cdot s + 3)^3 + K_c} = \frac{K_c}{(1000 \cdot s^3 + 900 \cdot s^2 + 270 \cdot s + 27 + K_c)}$$

using Routh-Hurwitz method,

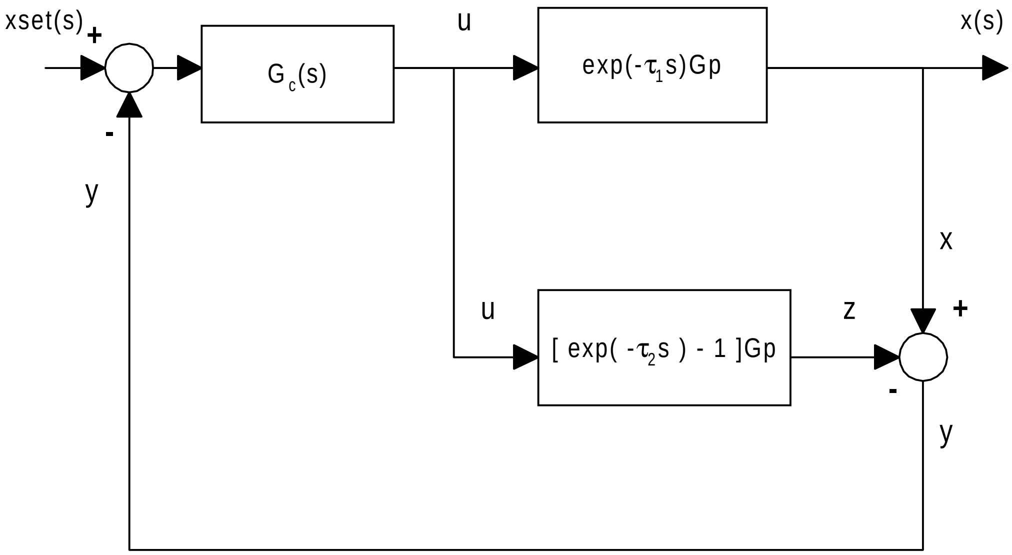
$$\begin{bmatrix} 1000 & 270 \\ 900 & 27 + K_c \\ 240 - \frac{10}{9} \cdot K_c & 0 \\ 27 + K_c & 0 \end{bmatrix}$$

ulitmate gain is obtained when

$$240 - \frac{10}{9} \cdot K_u = 0$$

$$K_u = 216$$

2.



$$x = e^{-\tau_1 \cdot s} \cdot G_p \cdot u$$

$$u = G_c \cdot (x_{set} - y)$$

$$y = x - z = x - \left(e^{-\tau_2 \cdot s} - 1\right) \cdot G_p \cdot u = \left(e^{-\tau_1 \cdot s} \cdot G_p - e^{-\tau_2 \cdot s} \cdot G_p + 1\right) \cdot u$$

$$u = G_c \cdot x_{set} - G_c \cdot \left(e^{-\tau_1 \cdot s} \cdot G_p - e^{-\tau_2 \cdot s} \cdot G_p + 1\right) \cdot u$$

$$u = \frac{G_c}{1 + G_c \cdot G_p \cdot \left(1 + e^{-\tau_1 \cdot s} - e^{-\tau_2 \cdot s}\right)} \cdot x_{set}$$

$$x = \left[\frac{G_c \cdot G_p \cdot e^{-\tau_1 \cdot s}}{1 + G_c \cdot G_p \cdot \left(1 + e^{-\tau_1 \cdot s} - e^{-\tau_2 \cdot s}\right)}\right] \cdot x_{set}$$

if $\tau_1 = \tau_2$

$$x = \left(\frac{G_c \cdot G_p \cdot e^{-\tau_1 \cdot s}}{1 + G_c \cdot G_p}\right) \cdot x_{set}$$

b)

$$G_{cl} = \frac{\left(\frac{-10\cdot s+1}{10\cdot s+1}\right)\cdot 2\cdot K_c}{50\cdot s+1+2\cdot K_c\cdot\left(1+\frac{-10\cdot s+1}{10\cdot s+1}-\frac{-11\cdot s+1}{11\cdot s+1}\right)}$$

$$G_{cl} = \frac{(-10\cdot s+1)\cdot\left[2\cdot(11\cdot s+1)\cdot K_c\right]}{\left(5500\cdot s^3+1160\cdot s^2+71\cdot s+1+220\cdot K_c\cdot s^2+46\cdot K_c\cdot s+2\cdot K_c\right)}$$

Characteristic Equation:

$$5500\cdot s^3+\left(220\cdot K_c+1160\right)\cdot s^2+\left(46\cdot K_c+71\right)\cdot s+2\cdot K_c+1=0$$

R-H Array:

$$\begin{bmatrix} 5500 & \left(46\cdot K_c+71\right) \\ \left(220\cdot K_c+1160\right) & \left(2\cdot K_c+1\right) \\ \frac{\left(506\cdot K_c{}^2+2899\cdot K_c+3843\right)}{\left(11\cdot K_c+58\right)} & 0 \\ \left(2\cdot K_c+1\right) & 0 \end{bmatrix}$$

For stability, $K_c > -0.5$

3.

$$G(s) = \frac{2\cdot s+1}{(s+1)\cdot\left[(s+2)^2+4\right]} = \frac{A}{s+1}+B\cdot\frac{s+2}{(s+2)^2+4}+C\cdot\frac{2}{(s+2)^2+4}$$

$$A = \lim_{s\to -1}\frac{2\cdot s+1}{\left[(s+2)^2+4\right]} = \frac{-1}{5}$$

$$Q = \lim_{s\to -2+2i}\frac{2\cdot s+1}{s+1} = \frac{11}{5}+\frac{2}{5}\cdot i$$

$$B = \frac{1}{2}\cdot\frac{2}{5} = \frac{1}{5}$$

$$C = \frac{1}{2}\cdot\frac{11}{5} = \frac{11}{10}$$

Check: $$\frac{-\frac{1}{5}}{s+1}+\frac{1}{5}\cdot\frac{s+2}{(s+2)^2+4}+\frac{11}{10}\cdot\frac{2}{(s+2)^2+4} = \frac{(2\cdot s+1)}{\left[(s+1)\cdot\left(s^2+4\cdot s+8\right)\right]}$$