## Solution to Exam 2, March 20, 2001

1. a) $$G_p(s)=\frac{1}{\left(\tau_1\cdot s+1+\tau_1\cdot k_1\right)\cdot\left(\tau_2\cdot s+1+\tau_2\cdot k_2\right)\cdot\left(\tau_3\cdot s+1+\tau_3\cdot k_3\right)}$$

b) $$G_{cl}(s)=\frac{K_c}{(10\cdot s+3)^3+K_c}=\frac{K_c}{\left(1000\cdot s^3+900\cdot s^2+270\cdot s+27+K_c\right)}$$

using Routh-Hurwitz method,

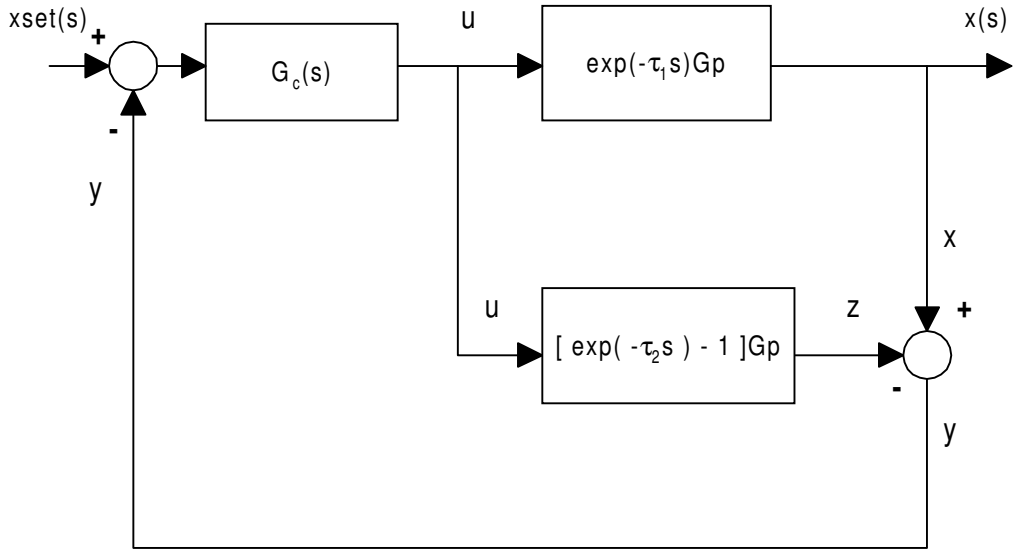
$$\begin{bmatrix} 1000 & 270 \\ 900 & 27+K_c \\ 240-\frac{10}{9}\cdot K_c & 0 \\ 27+K_c & 0 \end{bmatrix}$$

ulitmate gain is obtained when

$$240-\frac{10}{9}\cdot K_u=0$$

$$K_u=216$$

2.




$$x = e^{-\tau_1 \cdot s} \cdot G_p \cdot u$$

$$u = G_c \cdot \left( x_{set} - y \right)$$

$$y = x - z = x - \left( e^{-\tau_2 \cdot s} - 1 \right) \cdot G_p \cdot u = \left( e^{-\tau_1 \cdot s} \cdot G_p - e^{-\tau_2 \cdot s} \cdot G_p + 1 \right) \cdot u$$

$$u = G_c \cdot x_{set} - G_c \cdot \left( e^{-\tau_1 \cdot s} \cdot G_p - e^{-\tau_2 \cdot s} \cdot G_p + 1 \right) \cdot u$$

$$u = \frac{G_c}{1 + G_c \cdot G_p \cdot \left( 1 + e^{-\tau_1 \cdot s} - e^{-\tau_2 \cdot s} \right)} \cdot x_{set}$$

$$x = \left[ \frac{G_c \cdot G_p \cdot e^{-\tau_1 \cdot s}}{1 + G_c \cdot G_p \cdot \left( 1 + e^{-\tau_1 \cdot s} - e^{-\tau_2 \cdot s} \right)} \right] \cdot x_{set}$$

if $\tau_1 = \tau_2$

$$x = \left( \frac{G_c \cdot G_p \cdot e^{-\tau_1 \cdot s}}{1 + G_c \cdot G_p} \right) \cdot x_{set}$$

b) $$G_{cl}=\frac{\left(\frac{-10\cdot s+1}{10\cdot s+1}\right)\cdot 2\cdot K_c}{50\cdot s+1+2\cdot K_c\cdot\left(1+\frac{-10\cdot s+1}{10\cdot s+1}-\frac{-11\cdot s+1}{11\cdot s+1}\right)}$$

$$G_{cl}=\frac{(-10\cdot s+1)\cdot\left[2\cdot(11\cdot s+1)\cdot K_c\right]}{\left(5500\cdot s^3+1160\cdot s^2+71\cdot s+1+220\cdot K_c\cdot s^2+46\cdot K_c\cdot s+2\cdot K_c\right)}$$

Characteristic Equation:

$$5500\cdot s^3+\left(220\cdot K_c+1160\right)\cdot s^2+\left(46\cdot K_c+71\right)\cdot s+2\cdot K_c+1=0$$

R-H Array:

$$\begin{bmatrix} 5500 & \left(46\cdot K_c+71\right) \\ \left(220\cdot K_c+1160\right) & \left(2\cdot K_c+1\right) \\ \frac{\left(506\cdot K_c^{\,2}+2899\cdot K_c+3843\right)}{\left(11\cdot K_c+58\right)} & 0 \\ \left(2\cdot K_c+1\right) & 0 \end{bmatrix}$$

For stability, $K_c>-0.5$

3. $$G(s)=\frac{2\cdot s+1}{(s+1)\cdot\left[(s+2)^2+4\right]}=\frac{A}{s+1}+B\cdot\frac{s+2}{(s+2)^2+4}+C\cdot\frac{2}{(s+2)^2+4}$$

$$A=\lim_{s\to -1}\frac{2\cdot s+1}{\left[(s+2)^2+4\right]}=\frac{-1}{5}$$

$$Q=\lim_{s\to -2+2i}\frac{2\cdot s+1}{s+1}=\frac{11}{5}+\frac{2}{5}\cdot i$$

$$B=\frac{1}{2}\cdot\frac{2}{5}=\frac{1}{5}$$

$$C=\frac{1}{2}\cdot\frac{11}{5}=\frac{11}{10}$$

Check: $$\frac{-\frac{1}{5}}{s+1}+\frac{1}{5}\cdot\frac{s+2}{(s+2)^2+4}+\frac{11}{10}\cdot\frac{2}{(s+2)^2+4}=\frac{(2\cdot s+1)}{\left[(s+1)\cdot\left(s^2+4\cdot s+8\right)\right]}$$