Solution to Exam 2: March 28, 2002

1. Amplitude: $a := 0.015 \quad h := -0.20$

Ultimate Gain: $K_u := \frac{4}{\pi} \cdot \frac{h}{a} \qquad K_u = -16.977$

$P_u := 4.6$

PID Tuning:

$$K_c := \frac{K_u}{1.7} \qquad K_c = -9.986$$

$$\tau_I := \frac{P_u}{2} \qquad \tau_I = 2.3$$

$$\tau_D := \frac{P_u}{8} \qquad \tau_D = 0.575$$

2. $s \cdot L(P_1) = \alpha_1 \cdot L(P_0) - (\alpha_1 + \alpha_2) \cdot L(P_1) + \alpha_2 \cdot L(P_2)$

$s \cdot L(P_2) = \alpha_2 \cdot L(P_1) - (\alpha_2 + \alpha_3) \cdot L(P_2) + \alpha_3 \cdot L(P_{atm}) + \beta \cdot L(u)$

after reordering and letting $\alpha 1=2$, $\alpha 2=1$, $\alpha 3=2$, $\beta=3$,

$(s+3) \cdot L(P_1) = 2 \cdot L(P_0) + L(P_2)$

$(s+3) \cdot L(P_2) = L(P_1) + 2 \cdot L(P_{atm}) + 3 \cdot L(u)$

solving simulatneously for L(P1),

$$L(P_1) = \left(\frac{3}{s^2 + 6 \cdot s + 8}\right) \cdot L(u) + \left(\frac{2}{s^2 + 6 \cdot s + 8}\right) \cdot L(P_{atm}) + \left(\frac{2 \cdot s + 6}{s^2 + 6 \cdot s + 8}\right) \cdot L(P_0)$$

3.

$$L(y) = \left[\frac{E\cdot(A + MB)}{1 + E\cdot(A\cdot F + M\cdot B\cdot F - M\cdot C)} \right]\cdot L\left(y_{set}\right) + \left[\frac{D}{1 + E\cdot(A\cdot F + M\cdot B\cdot F - M\cdot C)} \right]\cdot L(d)$$

4. Closed loop transfer function:

$$G_{cl} = \frac{\left[\frac{K_c\cdot(10\cdot s + 1)\cdot(2\cdot s + 1)}{10\cdot s\cdot(0.1\cdot s + 1)} \right]\cdot\left(\frac{-2\cdot s + 1}{-3\cdot s + 1} \right)}{1 + \left[\frac{K_c\cdot(10\cdot s + 1)\cdot(2\cdot s + 1)}{10\cdot s\cdot(0.1\cdot s + 1)} \right]\cdot\left(\frac{-2\cdot s + 1}{-3\cdot s + 1} \right)}$$

$$= \frac{(2\cdot s + 1)\cdot\left[(2\cdot s - 1)\cdot K_c\cdot(10\cdot s + 1)\right]}{\left[\left(40\cdot K_c + 3\right)\cdot s^3 + \left(4\cdot K_c + 29\right)\cdot s^2 + \left(-10\cdot K_c - 10\right)\cdot s - K_c\right]}$$

Routh-Hurwitz Array:

$$\begin{bmatrix} 40\cdot K_c + 3 & -10\cdot K_c - 10 \\ 4\cdot K_c + 29 & -K_c \\ \frac{-\left(327\cdot K_c + 290\right)}{\left(4\cdot K_c + 29\right)} & 0 \\ -K_c & 0 \end{bmatrix}$$

No value of Kc is available that would make the first column have the same sign.

5. $$L(f) = \frac{-12\cdot s}{\left(s^2 + 9\right)^2}$$