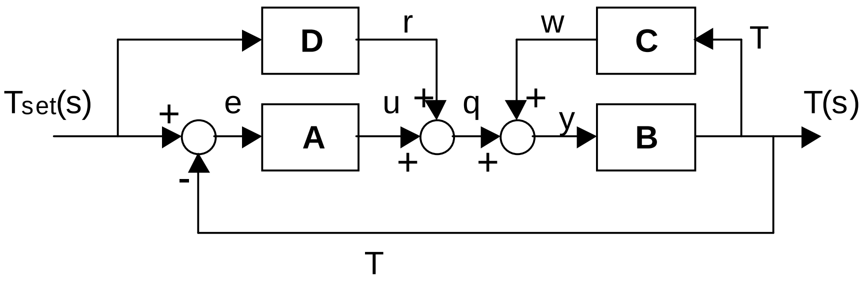
Solution to Exam 2. March 27, 2003

1. a)



$T = B \cdot y \qquad y = w + q \qquad w = C \cdot T$

$q = r + u \qquad r = D \cdot Tset \qquad u = A \cdot e \qquad e = Tset - T$

$$T = B \cdot (C \cdot T + D \cdot Tset + A \cdot e)$$

$$T = B \cdot C \cdot T + B \cdot D \cdot Tset + B \cdot A \cdot (Tset - T)$$

$$T = (B \cdot C - B \cdot A) \cdot T + (B \cdot D + B \cdot A) \cdot Tset$$

$$T = \left(\frac{B \cdot D + B \cdot A}{1 + B \cdot A - B \cdot C}\right) \cdot Tset$$

b) $A = \frac{K}{5 \cdot s + 1} \qquad B = \frac{1}{-3 \cdot s + 1} \qquad C = \frac{2}{s + 1} \qquad D = \frac{1 - A}{B} \cdot \left(\frac{1}{s + 1}\right)$

$$T = \left[\frac{\left[15 \cdot s^2 + (-2 - 4 \cdot K) \cdot s - 1\right]}{\left[15 \cdot s^3 + 13 \cdot s^2 + (-K + 7) \cdot s - K + 1\right]}\right] \cdot Tset$$

x

characteristic equation is given by:

$$15 \cdot s^3 + 13 \cdot s^2 + (7 - K) \cdot s + 1 - K = 0$$

Routh-Hurwitz array:

$$\begin{bmatrix} 15 & 7-K \\ 13 & 1-K \\ \frac{76}{13}+\frac{2}{13}\cdot K & 0 \\ 1-K & 0 \end{bmatrix}$$

Stable if: $-38<K<1$

2. $$\frac{d}{dt}C_{A1}=\frac{1}{\tau_1}\cdot\left(C_{Ain}-C_{A1}+\alpha\cdot C_{A2}\right)-k_1\cdot C_{A1}$$

$$\frac{d}{dt}C_{A2}=\frac{1}{\tau_2}\cdot\left(C_{A1}-C_{A2}\right)-k_2\cdot C_{A2}$$

$$s\cdot C_{A1}=\frac{1}{\tau_1}\cdot\left(C_{Ain}-C_{A1}+\alpha\cdot C_{A2}\right)-k_1\cdot C_{A1}$$

$$s\cdot C_{A2}=\frac{1}{\tau_2}\cdot\left(C_{A1}-C_{A2}\right)-k_2\cdot C_{A2}$$

$\tau_1 := 1$ $\quad\tau_2 := 2$ $\quad\alpha := 0.1$ $\quad k_1 := 0.2$ $\quad k_2 := 0.3$

$$C_{A2}(s)=\left[\frac{1}{\left(2\cdot s^2+4\cdot s+1.82\right)}\right]\cdot C_{Ain}(s)$$

$$C_{A2}(s)=\left[\frac{1}{(s+0.7)\cdot(s+1.3)}\right]\cdot\left(\frac{0.4}{s}+\frac{0.5}{s+1}\right)$$

$$C_{A2}(s)=\frac{0.9\cdot s+0.4}{s\cdot(s+1)\cdot(s+0.7)\cdot(s+1.3)}$$

$$C_{A2}(s)=\frac{A}{s}+\frac{B}{s+1}+\frac{C}{s+0.7}+\frac{D}{s+1.3}$$

$$A = \lim_{s \to 0} \frac{0.9 \cdot s + 0.4}{(s+1)\cdot(s+0.7)\cdot(s+1.3)} = 0.44$$

$$B = \lim_{s \to -1} \frac{0.9 \cdot s + 0.4}{s\cdot(s+0.7)\cdot(s+1.3)} = -5.556$$

$$C = \lim_{s \to -0.7} \frac{0.9 \cdot s + 0.4}{s\cdot(s+1)\cdot(s+1.3)} = 1.825$$

$$D = \lim_{s \to -1.3} \frac{0.9 \cdot s + 0.4}{s\cdot(s+1)\cdot(s+0.7)} = 3.291$$

$$C_{A2}(t) = 0.44 - 5.556\cdot e^{-t} + 1.825\cdot e^{-0.7\cdot t} + 3.291\cdot e^{-1.3\cdot t}$$

3. $$f = 5\cdot e^{-2\cdot t} - 2\cdot e^{-2\cdot t}\cdot \sin\left(\frac{2\cdot\pi}{10}\cdot t\right)$$

$$f(s) = \frac{5}{s+2} - 2\cdot\frac{\frac{2\cdot\pi}{10}}{(s+2)^2 + \left(\frac{2\cdot\pi}{10}\right)^2}$$