Solution for Exam2 March 18, 2004

1. Using the algebraic approach,

$$T = m + b = D\cdot d + B\cdot a = D\cdot d + B\cdot A\cdot(f - T)$$

$$(1 + A\cdot B)\cdot T = D\cdot d + B\cdot A\cdot f$$

$$(1 + A\cdot B)\cdot T = D\cdot d + B\cdot A\cdot G\cdot e = D\cdot d + B\cdot A\cdot G\cdot\left(T_{set} - w\right)$$

$$= D\cdot d + B\cdot A\cdot G\cdot T_{set} - B\cdot A\cdot G\cdot(T - q)$$

$$(1 + A\cdot B)\cdot T = D\cdot d + B\cdot A\cdot G\cdot T_{set} - B\cdot A\cdot G\cdot T + B\cdot A\cdot G\cdot Q\cdot f$$

(but : $B\cdot A\cdot f = (1 + A\cdot B)\cdot T - D\cdot d$)

$$(1 + A\cdot B)\cdot T = D\cdot d + B\cdot A\cdot G\cdot T_{set} - B\cdot A\cdot G\cdot T + G\cdot Q\cdot[(1 + A\cdot B)\cdot T - D\cdot d]$$

$$[(1 + A\cdot B)\cdot(1 - G\cdot Q) + B\cdot A\cdot G]\cdot T = D\cdot(1 - G\cdot Q)\cdot d + B\cdot A\cdot G\cdot T_{set}$$

$$T = \left[\frac{D\cdot(1 - G\cdot Q)}{(1 + A\cdot B)\cdot(1 - G\cdot Q) + B\cdot A\cdot G}\right]\cdot d + \left[\frac{B\cdot A\cdot G}{(1 + A\cdot B)\cdot(1 - G\cdot Q) + B\cdot A\cdot G}\right]\cdot T_{set}$$

2. After taking Laplace transform,

$$s\cdot C_A = -0.5\cdot C_A + 0.7\cdot C_B \qquad \text{(eqn 1)}$$

$$s\cdot C_B = 0.2\cdot C_A - 0.7\cdot C_B + C_{Bin} \qquad \text{(eqn 2)}$$

solve for C_B in eqn 1

$$C_B = \frac{1}{0.7}\cdot(s + 0.5)\cdot C_A$$

substitute into eqn 2,

$$\left[(s + 0.7)\cdot(s + 0.5)\cdot\frac{1}{0.7} - 0.2\right]\cdot C_A = C_{Bin}$$

$$C_A = \frac{70}{\left(100\cdot s^2 + 120\cdot s + 21\right)}\cdot C_{Bin}$$

3. $$G_{CL} = \frac{2\cdot K_c\cdot(2\cdot s + 1)\cdot(-2\cdot s + 1)}{(s + 1)^2\cdot(3\cdot s + 1) + 2\cdot K_c\cdot(2\cdot s + 1)\cdot(-2\cdot s + 1)}$$

characteristic equation: $3\cdot s^3 + \left(7 - 8\cdot K_c\right)\cdot s^2 + 5\cdot s + 1 + 2\cdot K_c = 0$

$$\begin{pmatrix} 3 & 5 \\ 7 - 8\cdot K_c & 1 + 2\cdot K_c \\ \dfrac{32 - 46\cdot K_c}{7 - 8\cdot K_c} & 0 \\ 1 + 2\cdot K_c & 0 \end{pmatrix}$$

For stability, we need: $-0.5 < K_c < 0.696$

4. $$G_{CL} = \frac{K_c\cdot(-s + 1)}{s^2 + \left(6 - K_c\right)\cdot s + \left(8 + K_c\right)}$$

ultimate gain will occur at: $K_c = K_u = 6$

at this value of Kc,: $s^2 + 14 = 0$

will have the following roots: $\sqrt{14}\cdot i, -\sqrt{-14}\cdot i$

so ultimate period is: $P_u := \dfrac{2\cdot\pi}{\sqrt{14}}$ $\qquad P_u = 1.679$

Using Ziegler-Nichols, $K_c := \dfrac{6}{2.2}$ $\qquad K_c = 2.727$

$\tau_I := \dfrac{P_u}{1.2}$ $\qquad \tau_I = 1.399$

5. $T_{ss} := 50\cdot\left(\dfrac{4}{8 + 4}\right)$

$e_{ss} := 50 - T_{ss}$ $\qquad e_{ss} = 33.333$