CM416 Third Exam
February 16, 1994
Open-Notes/Open-Books

Name:_____

Please answer each question briefly in the space provided.

1. **All-Pass Systems.** A transfer function, $G(s)$, is said to be all-pass if the magnitude $|G(i\omega)|$ is equal to 1 for all values of frequency ω. Let $y = 2x - u$ and

$$\frac{1}{\alpha}\frac{dx}{dt} = u - x \qquad ; \quad x(0) = 0$$

   (a) (10 pts) Show that the transfer function from u to y is given by

$$G(s) = \frac{\alpha - s}{\alpha + s}$$

   Answer:

   (b) (10 pts) Show that for this transfer function, $|G(i\omega)| = 1$ at all frequencies, and thus this is system is all-pass.
   Answer:

2. **Non-minimum phase 1st order lead.** (10 pts) A first order lead is nonminimum phase if the zero is in the right half plane. So for a 1st order system, the general nonminimum phase type is given by

$$G(s) = 1 \; - \; \tau_z s$$

Determine the value of $arg[G(i\omega)]$ as $\omega \longrightarrow \infty$.

Answer:

3. **Stability margins using Bode plots.** Suppose the plots in Figure 1 were obtained from the frequency response experiments as

(a) (10 pts) What is the phase margin ?
Answer:

(b) (10 pts) What is the gain margin ?
Answer:

4. **Complex Mapping Theorem.** (10 pts) Let $G(s)$ be given by

$$G(s) = \frac{s^2 - 2s + 1.16}{s^2 + 2s + 1.36}$$

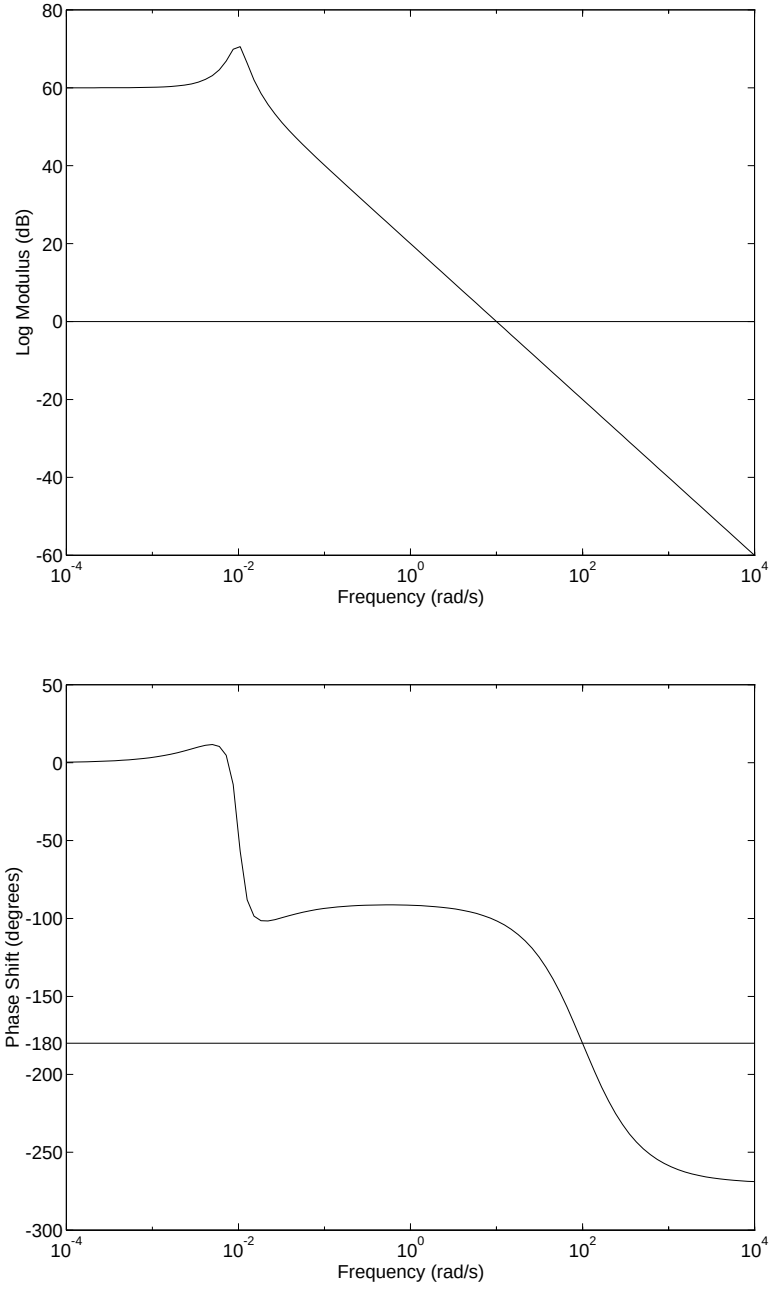

Figure 1: Bode plots of $G_c G_p$.

How many encirclements of the origin will the mapping $G(s)$ make as s traverses the contour $\Gamma(s)$ shown in Figure 2.

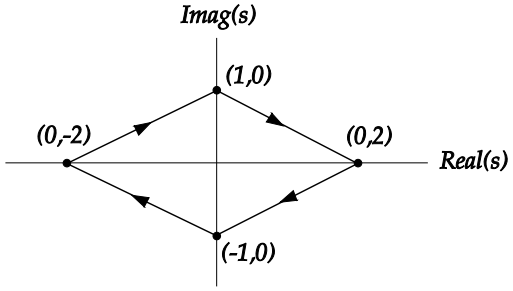

Figure 2: The contour $\Gamma(s)$.

Answer:

5. **Nyquist stability and robustness.** Suppose the Nyquist plot for G_p is shown in Figure 3.

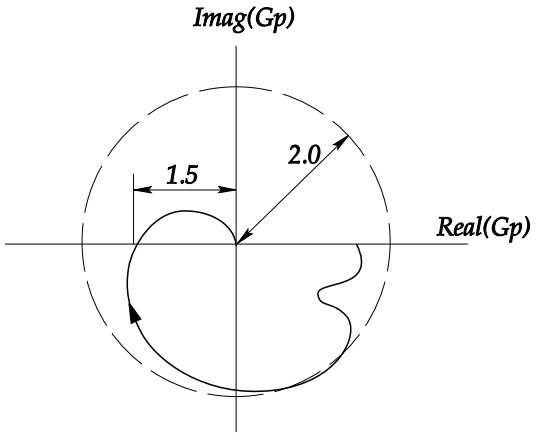

Figure 3: Nyquist plot of G_p.

(a) (10 pts) Using $K_c = 0.5$, will the feedback system, $K_c G_p/(1 + K_c G_p)$ be stable ? Why or why not ?
Answer:

(b) (10 pts) What maximum value of K_c should be used so that Log modulus of $K_c G_p$ is less than -2 dB for all frequencies ?
Answer:

6. **Proportional Control.** Suppose the plots shown in Figure 4 are Bode plots for the process, $G_p(s)$. Using proportional control,

   (a) (10 pts) What value of proportional contol gain is needed such that $K_c G_p$ will have a gain margin of 2.
   Answer:

   (b) (10 pts) What value of proportional control gain is needed such that $K_c G_p$ will have a phase margin of 45^o ?
   Answer:

   (c) (Bonus:10 pts) Which K_c value should be used? Why?
   Answer:

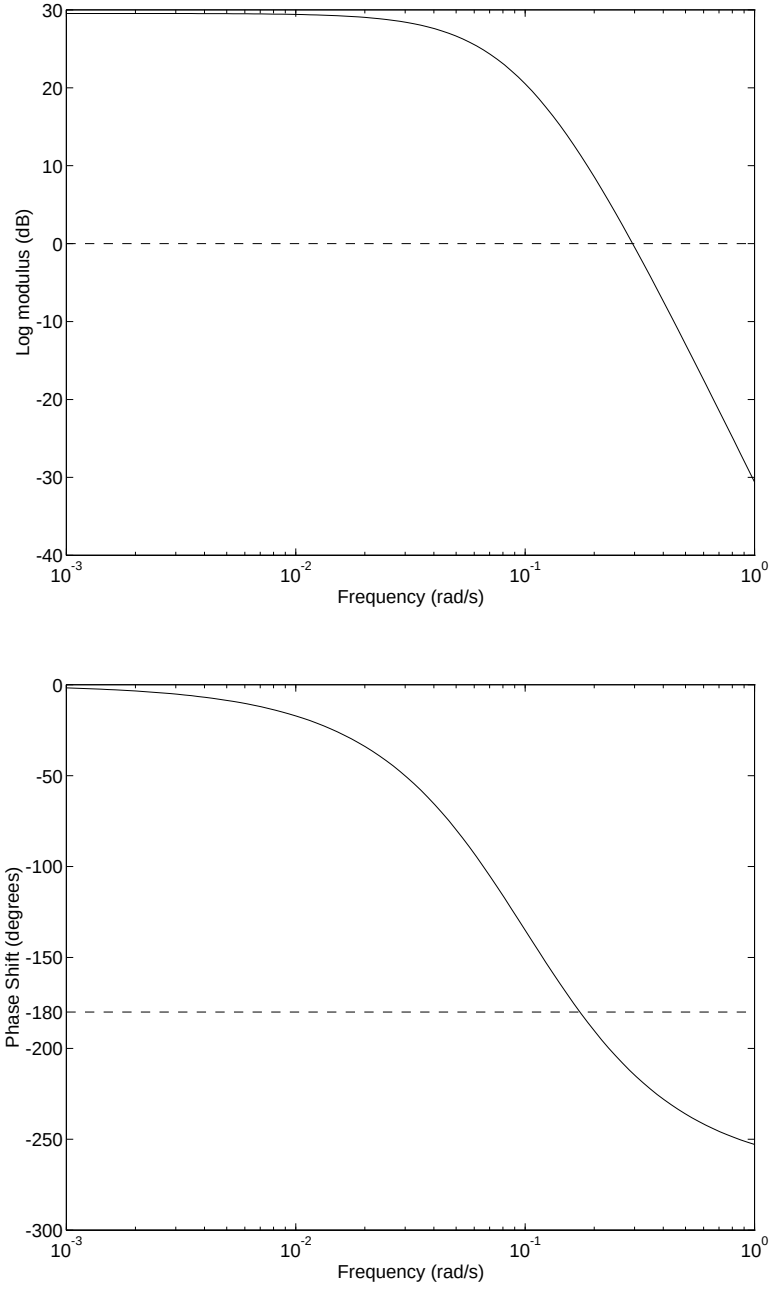

Figure 4: Bode plots of G_p.

Scratch Space:(Please still put answers in the spaces provided for each question.)