Exam 3. Feb. 15,1995
Open-Books/Open-Notes

Name:____

1. (10 pts) The Nyquist plot of $G_m(s)G_p(s)G_v(s)$ is shown in Figure 1 for the feedback control system shown in Figure 2. Using proportional control for G_c, what should the gain K_c be so that the system will have a gain margin of 3.

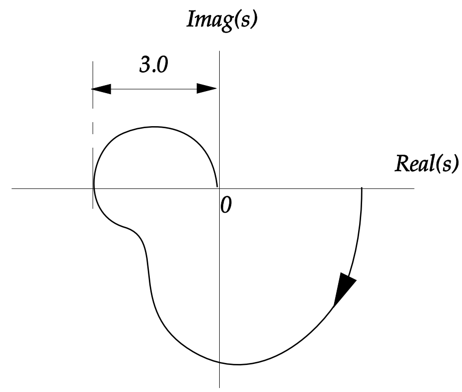

Figure 1: Feedback Control System.

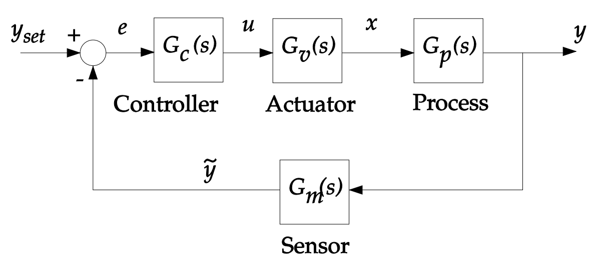

Figure 2: Feedback Control System.

2. In digital control system, the z-transform domain is used instead of the Laplace transform. The connection between the two domains lie in the definition of z as the complex map

$$z = e^{Ts}$$

where T is a fixed constant called the sampling period and s is the Laplace transform variable.

   (a) (10 pts) Obtain $|z|$ and $\arg(z)$ as s takes on values from 0 to $i\ 2\pi/T$ along the imaginary axis, where $i = \sqrt{-1}$.

   (b) (5 pts) Based on the previous answer, sketch the map of z as s takes on values from 0 to $i\ 2\pi/T$ along the imaginary axis.

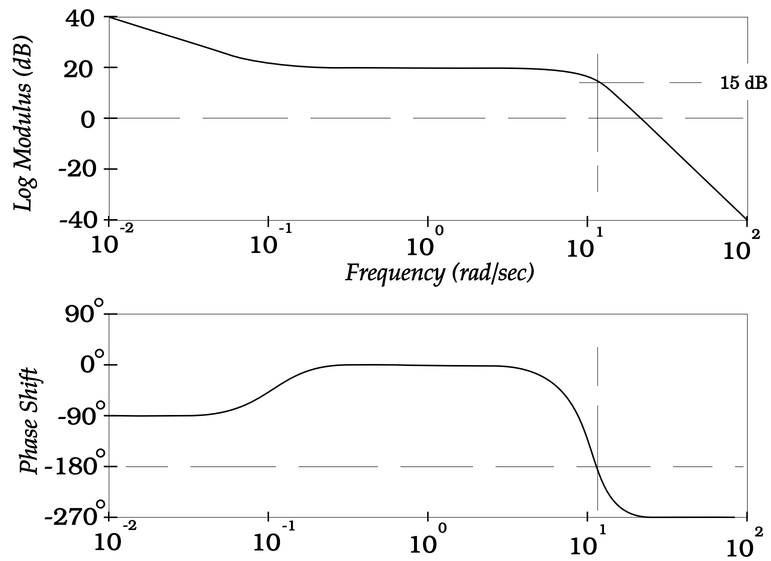

Figure 3: Feedback Control System.

3. The Bode plots for $G_m(s)G_p(s)G_v(s)$ are shown in Figure 3.

   (a) (10 pts) Using a proportional control, what should K_c be in order to obtain a gain margin of 10.

   (b) (10 pts) Using a proportional control with $K_c = 0.03$, what is the phase margin ?

4. (20 pts) Although the proportional control can stabilize several unstable systems, some systems can not be stabilized by the simple proportional control configuration. [1] Using Routh-Hurwitz method, determine whether the system in which

$$G_m(s)G_p(s)G_v(s) = \frac{s-1}{s^3+s^2-4s-4}$$

   can be stabilized by proportional control or not. If it can, what is the range of K_c which would stabilize the system?

5. (10 pts) Determine the gain and phase margins (in degrees) for a negative feedback control system whose Nyquist plot of $G_R(s) = G_cG_pG_vG_m(s)$is shown in Figure 4,

6. (15 pts) A frequency response experiment plot is shown in Figure 5. Determine the phase shift and magnitude ratio.

[1] One way to check stabilizability is to check what is known as the parity interlacing property. If there are an odd number of real positive poles between any real positive zeros (including ∞ if the numerator order is stricly less than the order of the denominator), then the process is not stabilizable by a stable controller, e.g. a proportional controller.

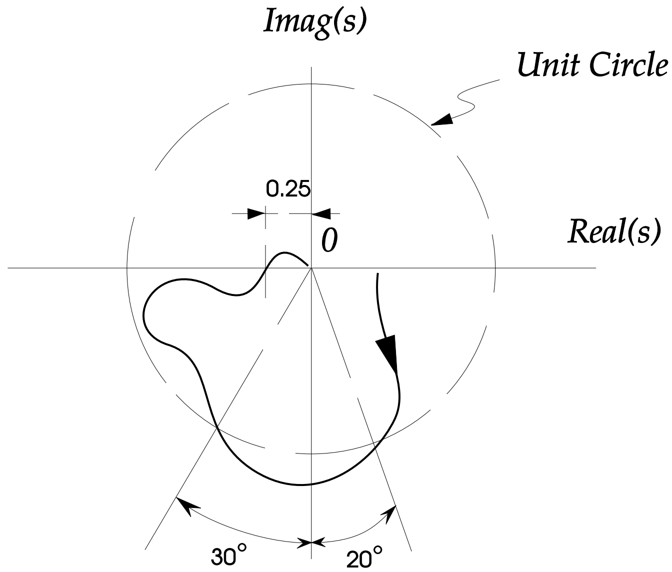

Figure 4: Feedback Control System.

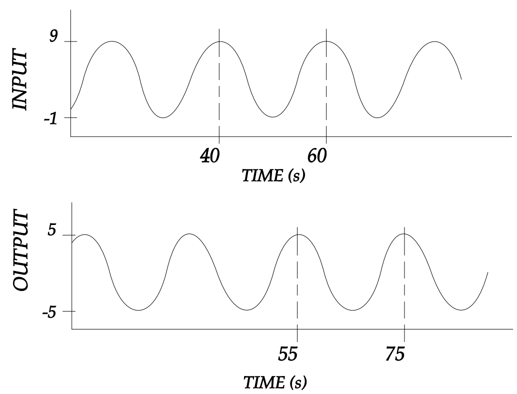

Figure 5: Frequency response data.

7. (10 pts) For the Bode plots shown in Figure 6 and 7, choose the transfer functions from Table 1 which would match the appropriate plots.

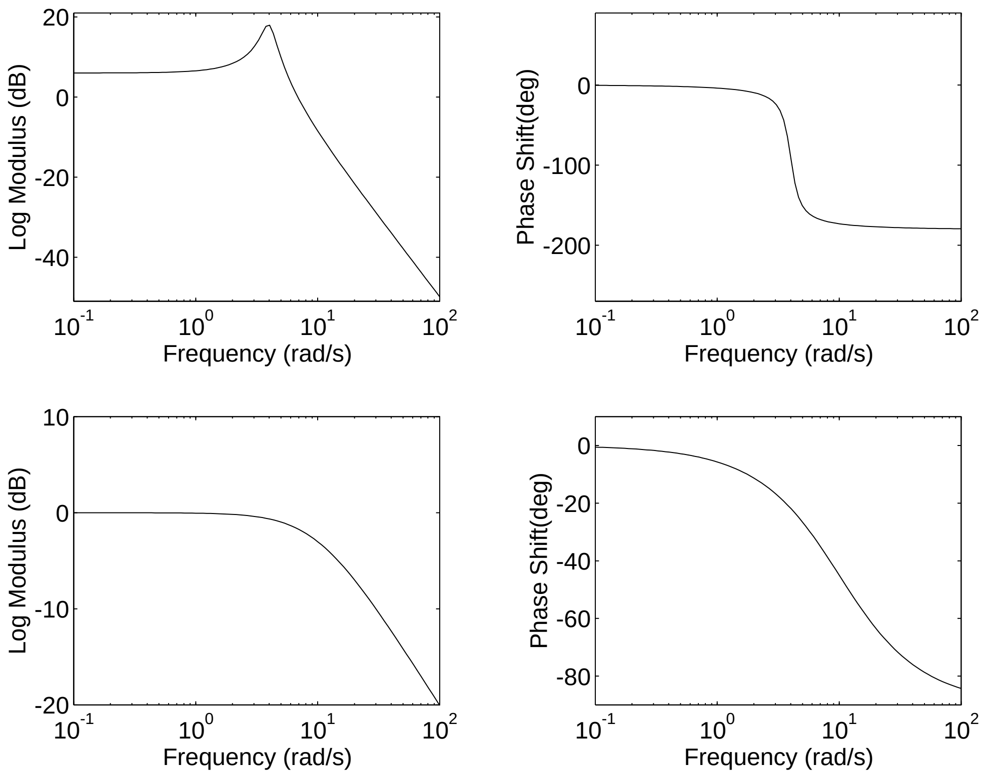

Figure 6: Bode plots for Case I (top pair), and Case II (bottom pair)

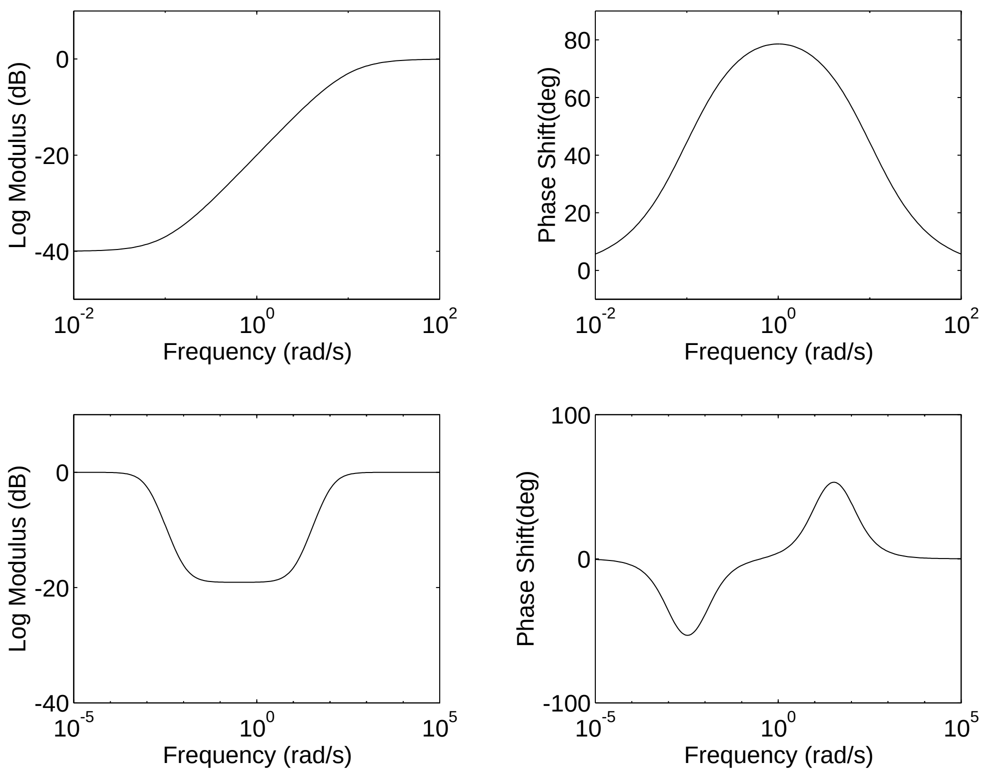

Figure 7: Bode plots for Case III (top pair), and Case IV (bottom pair)

Table 1: Collection of transfer functions for problem 7.

$G_1(s) =$	$\frac{10}{s+10}$
$G_2(s) =$	$\frac{1}{10s+1}$
$G_3(s) =$	$\frac{0.5s+1}{2s+1}$
$G_4(s) =$	$\frac{32}{s^2+s+16}$
$G_5(s) =$	$\frac{2}{4s^2+4s+1}$
$G_6(s) =$	$\frac{s}{s+10}$
$G_7(s) =$	$\frac{(0.09s+1)(100s+1)}{(900s+1)(0.01s+1)}$
$G_8(s) =$	$\frac{s+0.1}{s+10}$
$G_9(s) =$	$\frac{s+10}{s}$
$G_{10}(s) =$	$1 + \frac{1}{10s+1}$