

CM416 Exam 3
Open Books-Open Notes
February 16, 2000 7-9

Name: _____ Box No. _____

1. (20 pts) Although the Bode plots for an unstable process can not be obtained using frequency response experiments, one can still extend the notion of Bode plots for a given unstable transfer function $G(s)$ by still simply substituting $i\omega$ for s , and then using $|G(i\omega)|$ for amplitude ratio and $arg(G(i\omega))$ for phase shift. Comparing the Bode plots for a stable first order lag $G_1(s)$ with an unstable first order lag $G_2(s)$, where

$$G_1(s) = \frac{1}{\tau s + 1} \quad (\tau > 0)$$

$$G_2(s) = \frac{1}{-\tau s + 1}$$

show that the log modulus of both transfer functions are the same, i.e. $LM(G_1) = LM(G_2)$, but the phase shifts are negative of each other, i.e. $\phi(G_1) = -\phi(G_2)$.

2. (30 pts) From the feedback configuration shown in Figure 1, the Bode plots for the process G_p are shown in Figure 2, what would the tuning parameters of a PID controller be based on the Ziegler-Nichols method ?

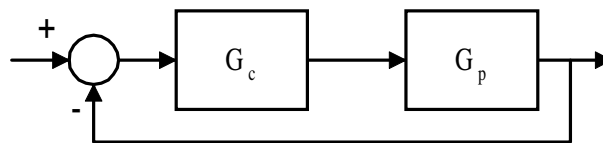


Figure 1.

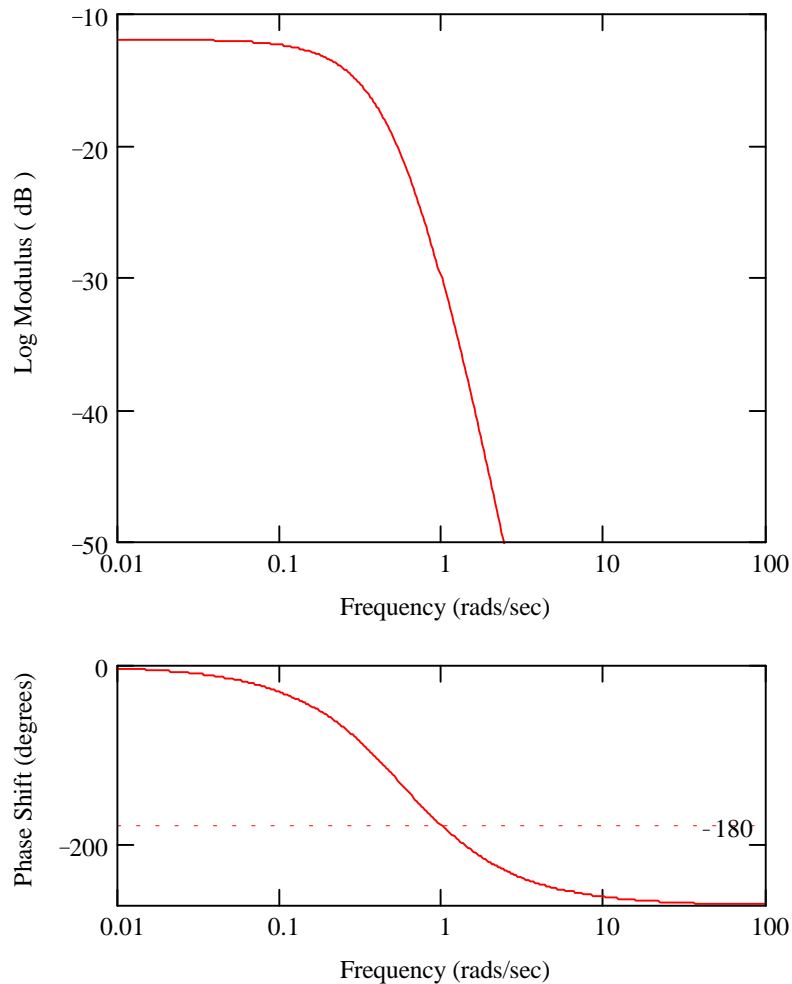


Figure 2. (dotted line shows -180° line).

3. Given the transfer function:

$$G(s) = \frac{K}{(\tau s + 1)^3} \quad (K > 0, \quad \tau > 0)$$

a) (15 pts) Show that the phase crossover frequency for $G(s)$ is:

$$\omega_{pc} = \frac{\sqrt{3}}{\tau}$$

(Hint: the phase crossover frequency for a third order lag is the nonzero, positive value of ω for which the imaginary part of $G(i\omega)$ is zero.)

(Additional Tip: Recall that $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$)

b) (15 pts) Using the crossover frequency in (a), show that the gain margin is given by: $GM=8/K$.

4. For the given cases in Table I, determine which transfer function from Table II matches the different cases.

Table I.	
Case 1	<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p>LM (dB)</p> <p>Frequency (rad/s)</p> </div> <div style="text-align: center;"> <p>Phase Shift (degrees)</p> <p>Frequency (rad/sec)</p> </div> </div>
Case 2	<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p>LM (dB)</p> <p>Frequency (rad/s)</p> </div> <div style="text-align: center;"> <p>Phase Shift (degrees)</p> <p>Frequency (rad/sec)</p> </div> </div>
Case 3	<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p>LM (dB)</p> <p>Frequency (rad/s)</p> </div> <div style="text-align: center;"> <p>Phase Shift (degrees)</p> <p>Frequency (rad/sec)</p> </div> </div>
Case 4	<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p>LM (dB)</p> <p>Frequency (rad/s)</p> </div> <div style="text-align: center;"> <p>Phase Shift (degrees)</p> <p>Frequency (rad/sec)</p> </div> </div>

Table II.	
G_1	$10 \cdot \frac{-s + 1}{s + 1}$
G_2	$\frac{10}{s + 10}$
G_3	$1 + \frac{1}{10 \cdot s}$
G_4	$10 \cdot \frac{0.1 \cdot s + 1}{10 \cdot s + 1}$
G_5	$\frac{10 \cdot s + 1}{(s + 1) \cdot (0.1 \cdot s + 1)}$
G_6	$10 \cdot \frac{s}{10 \cdot s + 1}$
G_7	0.01
G_8	$0.1 \cdot \left[\frac{(10 \cdot s + 1)}{10 \cdot s} \right] \cdot \left(\frac{0.1 \cdot s + 1}{0.001 \cdot s + 1} \right)$

5. (Bonus: 5pts) A sample plot of a frequency response experiment is shown in Figure 3, what is the phase shift (in degrees)?

