## CM3310

Third Exam
Open Book/Open Notes
April 15,2004 7-9pm

Name $\qquad$ Box No $\qquad$

1. (30 pts) For the feedback system shown in Figure 1, with the transfer functions given by

$$
\begin{aligned}
G_{p} & =\frac{5}{(2 s+1)} \\
G_{c} & =2 \frac{s+1}{s}
\end{aligned}
$$

obtain the magnitude ratio of $G_{\mathrm{cl}}$, the closed loop transfer function from $y_{\text {set }}$ to $y$, as a function of frequency, $\omega$ (rads/sec).


Figure 1.
(Hint/Check: Magnitude ratio of $G_{\mathrm{cl}}$ at $\omega=1 \mathrm{rad} / \mathrm{sec}$ is 1.04 )
2. (10 pts) Consider the same feedback structure shown in Figure 1, but with a different process transfer function, Gp, whose nyquist plot is shown in Figure 2. Using a proportional control, $G_{\mathrm{c}}=K_{\mathrm{c}}$, determine the value of $K_{\mathrm{c}}$ so that the resulting gain margin of $G_{\mathrm{c}} G_{\mathrm{p}}$ is 1.75 .


Figure 2. Nyquist plot of $G_{\mathrm{p}}$.
3. (30 pts) Consider again the feedback system shown in Figure 1, but this time the Bode plot of $G_{\mathrm{p}}$ is given in Figure 3. Obtain the PI control tuning based on the Tyreus-Luyben rules.


Figure 3.
4. (30 pts) Determine which of the transfer functions given in Table 1 matches the Bode plots shown in Figures 4, 5 and 6.

Table 1.

| G1 $=$ | $\frac{(10 s+1)(s+0.01)}{s(s+1000)}$ |  | G5 $=$ | $\frac{s+0.01}{s+100}$ |
| :---: | :---: | :---: | :---: | :---: |
|  | G2= | $\frac{10}{100 s^{2}+2 s+1}$ |  | $\frac{1}{100 s^{2}+2 s+1}$ |
| G3= | $\frac{s+100}{s+0.01}$ | G7 $=$ | $\frac{10(s+10)(s+0.01)}{s(s+1000)}$ |  |
| G4= | $1+\frac{-s+1}{0.01 s+1}$ |  | G8 $=$ | $\frac{e^{-2 s}}{s+100}$ |



Figure 4. Bode plot for Case 1.


Figure 5. Bode plot for Case 2.


Figure 6. Bode plot for Case 3.
5. Bonus ( 10 pts). Consider a circle contour $\Gamma$ of radius 1 and centered at $(-1,0)$ in the $s$ plane as shown in Figure 7. Find the number of clockwise encirclements of the origin that the map of

$$
G=\frac{s+1}{(s+1)^{2}+0.5^{2}}
$$

will have as $s$ traverses $\Gamma$ in the clockwise manner.


