Third Exam, February 16, 1994
Solution

1. a) Taking the Laplace transform of the differential equation:

$$\frac{1}{\alpha}\cdot s\cdot L(x) = L(u) - L(x)$$

$$L(x) = \left(\frac{\alpha}{s+\alpha}\right)\cdot L(u)$$

$$L(y) = 2\cdot L(x) - L(u) = \left[2\cdot\left(\frac{\alpha}{s+\alpha}\right) - 1\right]\cdot L(u)$$

$$. = \left[\frac{(\alpha - s)}{(s+\alpha)}\right]\cdot L(u)$$

Thus, $G = \dfrac{\alpha - s}{\alpha + s}$

b) $$G(i\cdot\omega) = \frac{(\alpha - i\cdot\omega)}{(\alpha + i\cdot\omega)} = \left[\frac{\alpha^2 - \omega^2}{\left(\alpha^2 + \omega^2\right)}\right] - i\cdot\frac{2\cdot\alpha\cdot\omega}{\left(\alpha^2+\omega^2\right)}$$

$$\left|G(i\cdot\omega)\right| = \sqrt{\left[\frac{\alpha^2 - \omega^2}{\left(\alpha^2+\omega^2\right)}\right]^2 + \left[\frac{2\cdot\alpha\cdot\omega}{\left(\alpha^2+\omega^2\right)}\right]^2}$$

$$. = \frac{1}{\left(\alpha^2+\omega^2\right)}\cdot\sqrt{\left(\alpha^2-\omega^2\right)^2 + 4\cdot\alpha^2\cdot\omega^2}$$

$$. = \frac{1}{\left(\alpha^2+\omega^2\right)}\cdot\sqrt{\alpha^4 - 2\cdot\alpha^2\cdot\omega^2 + \omega^4 + 4\cdot\alpha^2\cdot\omega^2}$$

$$. = \frac{1}{\left(\alpha^2+\omega^2\right)}\cdot\sqrt{\alpha^4 + 2\cdot\alpha^2\cdot\omega^2 + \omega^4}$$

$$. = \frac{1}{\left(\alpha^2+\omega^2\right)}\cdot\sqrt{\left(\alpha^2+\omega^2\right)^2}$$

$$. = 1$$

2. $G(i \cdot \omega) = 1 - \tau_z \cdot (i \cdot \omega)$ $\arg(G(i \cdot \omega)) = \operatorname{atan}(-\tau_z \cdot \omega) \cdot \frac{180}{\pi}$

$\operatorname{atan}(-\infty) = -90 \cdot deg$

3. $20 \cdot \log(x) = -20$

$x = 10^{-1}$

$GM = 10$

$PM = 180 - 100 = 80$

4. $zero = \begin{pmatrix} 1. + .4i \\ 1. - .4i \end{pmatrix}$ $poles = \begin{pmatrix} -1. + .6i \\ -1. - .6i \end{pmatrix}$

$Z = 2 \quad P = 0 \quad N = 2 - 0 = 2$

Thus there will be 2 clockwise encirclements of the origin.

5. a) with K_c=0.5, the magnitude ratio at -180º is 1.5*0.5=0.75. Thus K_cG_p will not encircle the point -1+0i, which implies that the feedback system will be stable.

b) $-2 = 20 \cdot \log(MR_{max})$

$MR_{max} = 10^{-\frac{2}{20}} = K_c \cdot 2$

$K_c = 10^{-\frac{2}{20}} \cdot \frac{1}{2} = 0.397$

6. a) Current $LM(G_p)$ @ 180^o = 11.67 dB

For GM=2, we want $LM(K_cG_p)$=20 log(1/2)

$$LM(K_c) = 20 \cdot \log\left(\frac{1}{2}\right) - LM(G_p) = -6.021 - 11.67 = -17.691 \cdot dB$$

$$K_c = 10^{\frac{-17.691}{20}} = 0.13$$

b) For a phase margin of 45^o, the corresponding LM(Gp) @ -135^o is +20dB.

$$LM(K_c) = -21dB$$

$$K_c = 10^{-\frac{21}{20}} = 0.09$$

c) use K_c=0.09, since this is the smaller of the two = more conservative setting.