Solution to Exam 3 February 16, 2000

1. $$G_1(i\cdot\omega) = \frac{1}{1 + i\cdot\tau\cdot\omega} = \frac{1}{\left(1+\tau^2\cdot\omega^2\right)} - i\cdot\tau\cdot\frac{\omega}{\left(1+\tau^2\cdot\omega^2\right)}$$

$$\left|G_1(i\cdot\omega)\right| = \sqrt{\left[\frac{1}{\left(1+\tau^2\cdot\omega^2\right)}\right]^2 + \left[\tau\cdot\frac{\omega}{\left(1+\tau^2\cdot\omega^2\right)}\right]^2}$$

$$\arg\left(G_1(i\cdot\omega)\right) = \operatorname{atan}(-\tau\cdot\omega)$$

$$G_2(i\cdot\omega) = \frac{1}{1 - i\cdot\tau\cdot\omega} = \frac{1}{\left(1+\tau^2\cdot\omega^2\right)} + i\cdot\tau\cdot\frac{\omega}{\left(1+\tau^2\cdot\omega^2\right)}$$

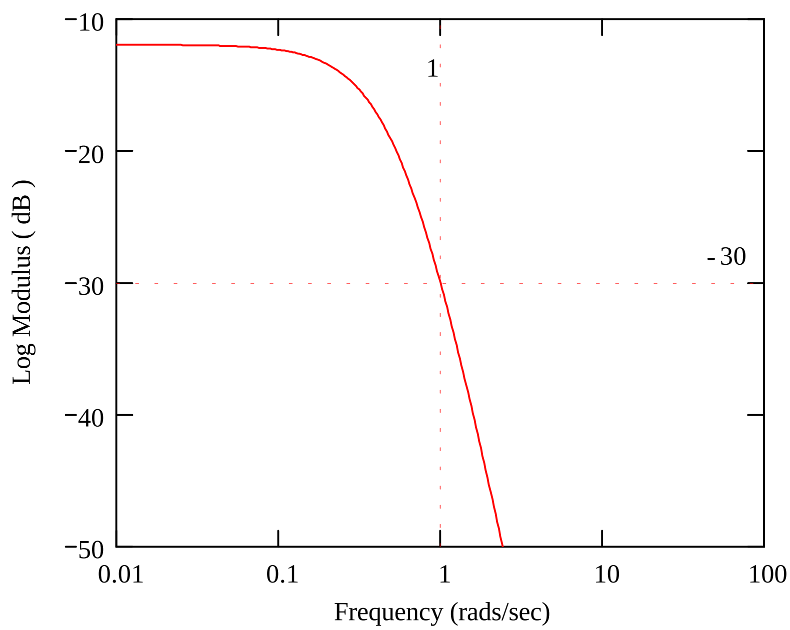
$$\left|G_2(i\cdot\omega)\right| = \sqrt{\left[\frac{1}{\left(1+\tau^2\cdot\omega^2\right)}\right]^2 + \left[\tau\cdot\frac{\omega}{\left(1+\tau^2\cdot\omega^2\right)}\right]^2}$$

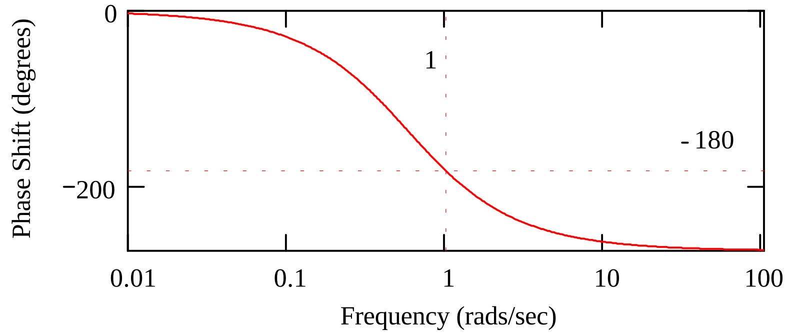
$$\arg\left(G_2(i\cdot\omega)\right) = \operatorname{atan}(\tau\cdot\omega)$$

since $\left|G_1(i\cdot\omega)\right| = \left|G_2(i\cdot\omega)\right|$ then $LM\left(G_1\right) = LM\left(G_2\right)$

since $\arg\left(G_1\right) = -\arg\left(G_2\right)$ then $\phi\left(G_1\right) = -\phi\left(G_2\right)$

2.





The phase crossover frequency is given by 1 rad/sec. Since the log modulus corresponding to this frequency is -30 dB, the ultimate gain is given by

$$20 \cdot \log(K_u) = 30$$

$$K_u := 10^{\frac{30}{20}} \qquad K_u = 31.623$$

while the corresponding ultimate period is given by

$$P_u := 2 \cdot \frac{\pi}{1} \cdot \left(\frac{sec}{cycle}\right) \qquad P_u = 6.283\ sec$$

Using Ziegler-Nichols tuning method, the controllers are given by,

$$K_c = \frac{K_u}{1.7} = \frac{31.623}{1.7} = 18.6$$

$$\tau_I = \frac{P_u}{2} = \frac{6.283}{2} = 3.14 \cdot sec$$

$$\tau_D = \frac{P_u}{8} = \frac{6.283}{8} = 0.785 \cdot sec$$

3.

$$G(i\cdot\omega)=\frac{K}{(1+i\cdot\tau\cdot\omega)^3}=\frac{K}{1+3\cdot(i\cdot\tau\cdot\omega)+3\cdot(i\cdot\tau\cdot\omega)^2+(i\cdot\tau\cdot\omega)^3}$$

$$G(i\cdot\omega)=\frac{K}{\left[1-3\cdot(\tau\cdot\omega)^2\right]+i\cdot\left[3\cdot\tau\cdot\omega-(\tau\cdot\omega)^3\right]}$$

$$=\frac{K\cdot\left[1-3\cdot(\tau\cdot\omega)^2\right]-i\cdot\left[3\cdot\tau\cdot\omega-(\tau\cdot\omega)^3\right]}{\left[1-3\cdot(\tau\cdot\omega)^2\right]^2+\left[3\cdot\tau\cdot\omega-(\tau\cdot\omega)^3\right]^2}$$

a) Phase crossover frequency: when Imag(G)=0

$$3\cdot\tau\cdot\omega-(\tau\cdot\omega)^3=0$$

$$\left[3-(\tau\cdot\omega)^2\right]\cdot(\tau\cdot\omega)=0$$

$$\omega=0,-\frac{\sqrt{3}}{\tau},\frac{\sqrt{3}}{\tau}$$

choosing positive value, $\omega_{pc}=\frac{\sqrt{3}}{\tau}$

b) The amplitude ratio at the crossover frequency is given by

$$AR_{(\omega_{pc})}=\left|G\left(i\cdot\frac{\sqrt{3}}{\tau}\right)\right|=\left|\frac{K\cdot\left[1-3\cdot\left(\tau\cdot\frac{\sqrt{3}}{\tau}\right)^2\right]-i\cdot\left[3\cdot\tau\cdot\frac{\sqrt{3}}{\tau}-\left(\tau\cdot\frac{\sqrt{3}}{\tau}\right)^3\right]}{\left[1-3\cdot\left(\tau\cdot\frac{\sqrt{3}}{\tau}\right)^2\right]^2+\left[3\cdot\tau\cdot\frac{\sqrt{3}}{\tau}-\left(\tau\cdot\frac{\sqrt{3}}{\tau}\right)^3\right]^2}\right|$$

$$AR_{(\omega_{pc})}=\left|\frac{K\cdot(1-9)}{(1-9)^2}\right|=\frac{K}{8}$$

Gain Margin: $GM=\frac{1}{AR_{(\omega_{pc})}}=\frac{8}{K}$

4. Case I: $G_6 = \frac{10 \cdot s}{10 \cdot s + 1}$

Case 2: $G_4 = 10 \cdot \frac{0.1 \cdot s + 1}{10 \cdot s + 1}$

Case 3: $G_1 = 10 \cdot \frac{1 - s}{1 + s}$

Case 4: $G_8 = 0.1 \cdot \left(\frac{10 \cdot s + 1}{10 \cdot s} \right) \cdot \left(\frac{0.1 \cdot s + 1}{0.001 \cdot s + 1} \right)$

5. $t_{shift} = -5$

$period = 10$

$\phi = -5 \cdot \frac{2 \cdot \pi}{10} = -\pi \cdot rads = -180^{o}$