

### Solution to Exam 3 April 17, 2001

1. Unstable if  $0.5 < K_c < 1.0$

2. a)  $LM(G_c) = 20 \cdot \log(0.4) = -7.96 \text{ dB}$

$$\omega_{pc} := 1 \frac{\text{rad}}{\text{sec}}$$

at  $\omega_{pc}$  :  $LM(G_p) = 10 \text{ dB}$

$$LM(G_c \cdot G_p) = (10 - 7.96) \text{ dB} = 2.04 \text{ dB} > 0 \text{ dB}$$

Thus not stabilized.

b)  $K_u := 10^{-\frac{10}{20}}$        $K_u = 0.316$

$$P_u := \frac{2 \cdot \pi}{1}$$
       $P_u = 6.283$

$$K_c := \frac{K_u}{1.7}$$
       $K_c = 0.186$

$$\tau_I := \frac{P_u}{2}$$
       $\tau_I = 3.142$

$$\tau_D := \frac{P_u}{8}$$
       $\tau_D = 0.785$

3.  $G(s) = \frac{1}{s+1} - \frac{2}{s+3} = \frac{(-s+1)}{(s+1) \cdot (s+3)}$

$$MR(\omega) = \frac{1}{3} \cdot \frac{\sqrt{1+\omega^2}}{\sqrt{1+\omega^2} \cdot \sqrt{1+\frac{\omega^2}{9}}} = \frac{1}{\sqrt{9+\omega^2}}$$

4. From Figure 5, phase crossover frequency is  $\omega_{pc} := 10 \cdot \frac{\text{rad}}{\text{sec}}$

while the gain crossover frequency is  $\omega_{gc} := 6.1 \cdot \frac{\text{rad}}{\text{sec}}$

Gain Margin : Let x be MR @  $\omega_{pc}$

$$20 \cdot \log(x) = -10 \cdot \text{db}$$

$$x := 10^{-\frac{10}{20}} \quad x = 0.316$$

$$\text{GM} := \frac{1}{x} \quad \text{GM} = 3.162$$

Phase Margin: Phase shift @  $\omega_{gc}$   $\phi := -130 \cdot \text{deg}$

$$\text{PM} := 180 \cdot \text{deg} + \phi \quad \text{PM} = 50 \cdot \text{deg}$$

5. case 1: G7  
case 2: G4  
case 3: G1  
case 4: G5

6.  $\omega := \frac{2 \cdot \pi \cdot \text{rad}}{10 \cdot \text{sec}} \quad \omega = 0.628 \text{ sec}^{-1}$

$$\phi := (-8 \cdot \text{sec}) \cdot \omega \cdot \frac{180 \cdot \text{deg}}{\pi} \quad \phi = -288 \cdot \text{deg}$$

$$\text{MR} := \frac{20}{10}$$

$$\text{LM} = 20 \cdot \log(\text{MR}) = 6.021 \cdot \text{dB}$$