## Linearization:

1. $\frac{d}{dt}x = u \cdot \sin\left(\frac{\pi}{2} \cdot x\right) + 5$

operating point: $u_{op} = 2$ $x_{op} = 1$

Linearized equation:

$$\frac{d}{dt}x = \left(u_{op} \cdot \sin\left(\frac{\pi}{2} \cdot x_{op}\right) + 5\right) + \left(u_{op} \cdot \cos\left(\frac{\pi}{2} \cdot x_{op}\right) \cdot \frac{\pi}{2}\right) \cdot \left(x - x_{op}\right) + \sin\left(\frac{\pi}{2} \cdot x_{op}\right) \cdot \left(u - u_{op}\right)$$

$$\frac{d}{dt}x = 5 + u$$

2. $\frac{d}{dt}x = -x \cdot y + 5$

$$\frac{d}{dt}y = 3 \cdot x^2 + 6 \cdot y$$

operating point: steady state

$$0 = -x_{ss} \cdot y_{ss} + 5$$

$$0 = 3 \cdot x_{ss}^{\ 2} + 6 \cdot y_{ss}$$

solving for steady states: $x_{ss} = 10^{\frac{1}{3}}$ $y_{ss} = -\frac{1}{2} \cdot 10^{\frac{2}{3}}$

linearized system:

$$\frac{d}{dt}x = -y_{ss} \cdot \left(x - x_{ss}\right) - x_{ss} \cdot \left(y - y_{ss}\right)$$

$$\frac{d}{dt}y = \left(6 \cdot x_{ss}\right) \cdot \left(x - x_{ss}\right) + 6 \cdot \left(y - y_{ss}\right)$$

or

$$\frac{d}{dt}x = \frac{1}{2} \cdot \left(\sqrt[3]{10}\right)^2 \cdot x - 10 - \sqrt[3]{10} \cdot y$$

$$\frac{d}{dt}y = 6 \cdot \sqrt[3]{10} \cdot x - 3 \cdot \left(\sqrt[3]{10}\right)^2 + 6 \cdot y$$

3. Given: $\frac{d}{dt}z = 5 \cdot z - z \cdot \sqrt{u}$

and the linearized equation is given by: $\frac{d}{dt}z = z + u + \alpha$

The general linearized equation around the operating points are:

$$\frac{d}{dt}z = \left(5 \cdot z_{op} - z_{op} \cdot \sqrt{u_{op}}\right) + \left(5 - \sqrt{u_{op}}\right) \cdot \left(z - z_{op}\right) + \left(\frac{z_{op}}{2 \cdot \sqrt{u_{op}}}\right) \cdot \left(u - u_{op}\right)$$

to match given linearized equation,

$$5 - \sqrt{u_{op}} = 1 \qquad \text{or} \qquad u_{op} = 16$$

$$\frac{z_{op}}{2 \cdot \sqrt{u_{op}}} = 1 \qquad z_{op} = 8$$

thus,

$$\frac{d}{dt}z = -16 + z + u$$

or

$$\alpha = -16$$