Additional Exercises for Solution to ODEs

1. Given

$$\begin{aligned} \frac{dx}{dt} + ax &= 3e^{-2t} \\ x(0) &= 2 \end{aligned}$$

   (a) Solve the differential equation and plot $x(t)$ for $a = -1, 1, 2$

   (b) Determine the stability and steady states

2. Given

$$\begin{aligned} \frac{d^2y}{dt^2} + 5y &= -3\frac{dy}{dt} + 2 \\ y(0) &= 1 \\ \frac{dy}{dt}(0) &= \frac{1}{2} \end{aligned}$$

   (a) Solve the differential equation and plot $y(t)$

   (b) Determine the stability and steady states

3. Given

$$\begin{aligned} \frac{dx}{dt} &= -3x + 2y + 3 \\ \frac{dy}{dt} + 5y &= 2x - 4 \\ x(0) &= 1 \\ y(0) &= -1 \end{aligned}$$

   (a) Solve the set of differential equations and plot $x(t)$ and $y(t)$

   (b) Determine the stability and steady states

4. Given

$$\begin{aligned} \frac{d^2z}{dt^2} + 4z &= \cos(2t) \\ z(0) &- 1 \\ \frac{dz}{dt}(0) &= 0 \end{aligned}$$

   (a) Solve the set of differential equations and plot $z(t)$

   (b) Determine the stability and steady states

You could also simulate and plot the above systems using numerical methods and then compare with the plots obtained from the analytical solutions you found.