CM4110
PRESENTATION OF SAMPLE CALCULATIONS
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OBJECTIVE

The objective of the Sample Calculations is to document how the experimental results have been calculated from the raw data. Well presented sample calculations will establish the author's credibility with the reader. Other than units conversion factors, the source of *every* number must be documented, either by referring to a data table or to a literature reference.

The following principles should be followed:

1. BODY OF REPORT
   Whenever an experimental result is presented in the body of the report, there should be a reference to the corresponding appendix that contains the sample calculation leading up to that result.
2. TABLE OF NOMENCLATURE
   All variables, including those used in the sample calculations, must be listed in the Table of Nomenclature.
3. REFERENCES
   All literature sources, including those used in the sample calculations, must be listed in the References.
4. APPENDICES
   At least one appendix should contain the raw data. Use a sufficient number of appendices for raw data or subdivide a single appendix for raw data in such a manner that the reader knows exactly where each type of raw data is located. Make appropriate references in the body of the report to the Appendix(ices) containing the raw data.

   At least one appendix should contain the sample calculations. Again, use a sufficient number of appendices for sample calculations or subdivide a single appendix for sample calculations in such a manner that the reader knows exactly where each type of experimental result is calculated. The sample calculation should start with the equation to be used (where appropriate, refer to an equation in the body of the report or to the literature source of the equation). Then, present the value of each number to be substituted into the equation and an appropriate reference. Finally, show the substitutions into the equation and all units conversions.

EXAMPLE

The following pages demonstrate the above principles for the calibration of the capillary viscometer in the non-Newtonian flow experiment.

(Body of Report)

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BACKGROUND/THEORY

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The calibration equation for determining the capillary diameter using an aqueous glycerol solution is (Pintar, 1997):

$$\mathbf{D^4 = \frac{128\mu_{GL}LQ}{\pi g_c[(p_i - p_o) + \frac{g}{g_c}\rho_{GL}(h_i - h_o)]}} \qquad \text{(Eq. 5)}$$

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RESULTS

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The raw data for calibration of the capillary viscometer using an aqueous glycerol solution are contained in Appendix A. These data were used to determine the “dynamic” capillary diameter (Appendix D). The final result for the “dynamic” capillary diameter is 0.0397±0.00036 inches.

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TABLE OF NOMENCLATURE

D = "Dynamic" capillary diameter, in or ft
e_{Rx} = Reading error associated with the variable x
e_x = Standard error of the variable x
g = Acceleration of gravity, ft/s^2 = 32.174 ft/s^2
g_c = Newton's Law Conversion Factor, ft-lb/(lb_f-s^2) = 32.174 ft-lb/(lb_f-s^2)
h_i = Height of liquid level in feed vessel, in or ft
h_o = Height of liquid level at capillary discharge, in or ft
Δh = Manometer reading, in or ft
K = Flow consistency index, kg/[m-$s^{(2-n)}$] or lb/[ft-$s^{(2-n)}$]
L = Length of capillary tube, in or ft
M = Mass of liquid collected from capillary, g, kg, or lb
M_{GL} = Mass of glycerol solution plus pycnometer, g
M_P = Mass of pycnometer, g
M_W = Mass of water plus pycnometer, g
m = mass flow rate of liquid from capillary tube, kg/s or lb/s
n = Flow behavior index, dimensionless
p_i = Pressure at liquid level in feed vessel, Pa or lb_f/ft^2
p_o = Pressure at capillary discharge, Pa or lb_f/ft^2
Q = Volumetric flow rate of liquid from capillary tube, m^3/s or ft^3/s
S_w = Shear rate at the wall, s^{-1}
s_x = Standard deviation of any variable x
T = Temperature, °C
Δt = Time increment for flow rate measurement, s
V_P = Volume of pycnometer, cm^3

<u>Greeks</u>

μ_{GL} = Viscosity of glycerol solution, cP, Pa-s, or lb/(ft-s)
ρ_{GL} = Density of glycerol solution, lb/ft^3, g/cm^3, or kg/m^3
ρ_m = Density of manometer fluid, lb/ft^3, g/cm^3, or kg/m^3
ρ_W = Density of liquid water, lb/ft^3, g/cm^3, or kg/m^3
τ_w = Shear stress at the wall, lb_f/ft^2 or Pa (N/m^2)

REFERENCES

Bird, R.B., Stewart, W.E., and Lightfoot, E.N., *Transport Phenomena*, Wiley (1960), pp. 67-8.

Daniels, F., Matthews, J. H., Williams, J. W., Bender, P., and Alberty, R. A., *Experimental Physical Chemistry*, 5th Ed., McGraw-Hill (1956), pp. 375-6.

Dean, J.A. (editor), *Lange's Handbook of Chemistry*, 11th Ed., McGraw-Hill, NY (1973), pp. 10-94 to 10-96, 10-125, and 10-289.

McCabe, W.L., Smith, J.C., and Harriott, P., *Unit Operations of Chemical Engineering*, 5th Ed., McGraw-Hill (1993), pp. 48-9.

Pintar, A.J., "Capillary Viscometer Equations For a Power Law Fluid," Unit Operations Laboratory, Michigan Technological University, Houghton, MI (1997), p. 2.

Pintar, A.J., "Error Analysis," Unit Operations Laboratory, Michigan Technological University, Houghton, MI (2001).

APPENDIX A
RAW DATA FOR CALIBRATION OF CAPILLARY VISCOMETER

Table A-1
Pycnometer Data for Density of Glycerol Solution
(Temperature = 26.4 °C)

	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6
Mass of Pycnometer, M_P (g)	12.5380	12.5388	12.7648	12.7647	12.7647	12.5409
Mass of Water plus Pycnometer, M_W (g)	22.6460	22.6456	22.9676	22.9678	22.9678	22.6474
Mass of Glycerol Solution plus Pycnometer, M_{GL} (g)	24.7575	24.7582	25.0926	25.0941	25.0941	24.7555

Table A-2
Capillary Viscometer Data for Glycerol Solution
[Capillary Length (L) = 24 in]

Hg Manometer Reading, Δh (cm)	8.70	15.05	16.50	22.40	32.72	36.20	44.00
Liquid Level in Feed Vessel, h_i-h_o (in)	-2.4375	-2.4375	-3.9375	-3.24	-3.375	-2.4375	-3.50
Mass Collected, M (g)	4.575	13.70	14.26	7.100	12.465	10.28	12.89
Collection Time, Δt (sec)	300	600	600	180	300	180	180
Temperature (°C)	26.4	25.3	25.1	27.0	24.8	24.6	25.9

APPENDIX D

SAMPLE CALCULATIONS FOR CALIBRATION OF CAPILLARY VISCOMETER

D.1. Calculation of Density of Glycerol Solution

For calibration of the pycnometer with water (Daniels *et al*, 1956):

$$\mathbf{V_P = \frac{[M_W - M_P]}{\rho_W}} \quad \text{(Eq. D-1)}$$

The density of the glycerol solution is given by:

$$\mathbf{\rho_{GL} = \frac{[M_{GL} - M_P]}{V_P} = \frac{[M_{GL} - M_P]}{[M_W - M_P]}\rho_W} . \quad \text{(Eq. D-2)}$$

From Table A-1 (Appendix A) for Trial #1:

M_{GL} = 24.7575 g M_W = 26.646 g M_P = 12.5380 g

ρ_W(26.4 °C) = 0.99668 g/cm^3 (Dean, 1973, p. 10-127)

$$\mathbf{\rho_{GL} = \frac{[24.7575 - 12.5380]\ g}{[22.646 - 12.5380]\ g}(0.99668)\frac{g}{cm^3} = 1.2049\ g/cm^3} .$$

The other five trials in Table A-1 (Appendix A) were used to calculate the density in the same way as above. The six values for the density can be averaged and the standard deviation determined (Pintar, 2001):

$$\mathbf{\bar{\rho}_{GL} = \frac{\sum_{i=1}^{n}(\rho_{GL})_i}{n} = \frac{1.2049 + 1.2050 + 1.2043 + 1.2044 + 1.2044 + 1.2046}{6} = 1.2046\ \frac{g}{cm^3}}$$

$$\mathbf{s_{\rho_{GL}} = \sqrt{\frac{\sum_{i=1}^{n}[(\rho_{GL})_i - \bar{\rho}_{GL}]^2}{(n-1)}} = \sqrt{\frac{(9+16+9+4+4+0)x10^{-8}}{5}} = 0.00065\ \frac{g}{cm^3}} .$$

The reading error for each of the masses in Table A-1 (Appendix A) is ±0.0001 g and the reading error for the density of water is taken to be 0 g/cm^3. From the equation for the density of the glycerol solution (above), the reading error associated with ρ_{GL} is given by:

$$\mathbf{e_{R\rho_{GL}} = \frac{[\bar{M}_{GL} - \bar{M}_P]}{[\bar{M}_W - \bar{M}_P]}\rho_W\left[\frac{(e_{M_{GL}} + e_{M_P})}{(\bar{M}_{GL} - \bar{M}_P)} + \frac{(e_{M_W} + e_{M_P})}{(\bar{M}_W - \bar{M}_P)}\right]} . \quad \text{(Eq. D-3)}$$

Averaging the values in Table A-1 (Appendix A) and substituting into Eq. D-3:

$$\mathbf{e_{R\rho_{GL}} = \frac{[24.9253 - 12.6320]}{[22.8070 - 12.6320]}(0.99688)\frac{g}{cm^3}\left[\frac{(2)(0.0001)}{(24.9253 - 12.6320)} + \frac{(2)(0.0001)}{(22.8070 - 12.6320)}\right]}$$

$$\mathbf{e_{R\rho_{GL}} = 0.000043\ \frac{g}{cm^3}} .$$

Since the reading error is much smaller than the standard deviation, the error in the density of the glycerol solution is (Pintar, 1998):

$$\mathbf{e_{\rho GL} = \frac{s_{\rho GL}}{\sqrt{n}} = \frac{0.00065}{\sqrt{6}}\ \frac{g}{cm^3} = 0.00027\ \frac{g}{cm^3}}.$$

Thus, the density of the glycerol solution is:

$$\mathbf{\rho_{GL} = 1.2046 \pm 0.00027\ \frac{g}{cm^3}}.$$

D.2. Viscosity of Glycerol Solution

The density of the glycerol solution can be used to get the concentration of glycerol in the solution and then the viscosity of the glycerol solution:

79.0 wt-% glycerol @ ρ_{GL} = 1.2046 g/cm^3 (Dean, 1973, p. 10-96)

μ_{GL} = 38.49 cP (Interpolated from Dean, 1973, p. 10-289) @ 26.4 °C.

D.3. Calculation of Pressure Drop Across Capillary

From the manometer equation for a gas in the manometer legs (McCabe *et al.*, 1993, p. 33):

$$\mathbf{p_i - p_o = \frac{g}{g_c}\rho_m \Delta h} \qquad \text{(Eq. D-4)}$$

For a manometer reading of 8.70 cm Hg (Appendix A, Table A-2):

ρ_m = 13.5312 g/cm^3 for Hg @ 26.4 °C (Dean, 1973, p. 10-125)
Δh = 8.70 cm

$$\mathbf{p_i - p_o = \frac{(32.174)\frac{ft}{s^2}}{(32.174)\frac{ft-lb}{lb_f - s^2}}(13.5312)(62.4)\frac{lb}{ft^3}(8.70)cm\frac{(1)ft}{(30.48)cm} = 241\ lb_f / ft^2}.$$

For the total pressure drop, $\mathbf{[(p_i - p_o) + \frac{g}{g_c}\rho_{GL}(h_i - h_o)]}$ (Pintar, 2001):

ρ_{GL} = 1.2046 g/cm^3 (D.1 above) $\qquad$ $\mathbf{(h_i - h_o)}$ = -2.4375 in (Table A-2)

$$\mathbf{[(p_i - p_o) + \frac{g}{g_c}\rho_{GL}(h_i - h_o)] = 241\frac{lb_f}{ft^2} + (1)\frac{lb_f}{lb}(1.2046)(62.4)\frac{lb}{ft^3}(-2.4375)in\frac{(1)ft}{(12)in}}$$

$$\mathbf{[(p_i - p_o) + \frac{g}{g_c}\rho_{GL}(h_i - h_o)] = 226\frac{lb_f}{ft^2}}.$$

D.4. Calculation of Volumetric Flow Rate from Capillary

$$\mathbf{Q = \frac{m}{\rho_{GL}} = \frac{M}{\rho_{GL}\Delta t}} \qquad \text{(Eq. D-5)}$$

For a manometer reading of 8.70 cm Hg (Appendix A, Table A-2):

M = 4.575 g   Δt = 300 s ρ_{GL} = 1.2046 g/cm^3 (See Sec. D.1)

$$\mathbf{Q = \frac{(4.575)g\dfrac{(1)ft^3}{(30.48)^3cm^3}}{(1.2046)\dfrac{g}{cm^3}(300)s} = 4.471x10^{-7}\frac{ft^3}{s}}.$$

D.5. Calculation of “Dynamic” Capillary Diameter

The equation for the “dynamic” capillary diameter was given earlier (Eq. 5):

$$\mathbf{D^4 = \frac{128\mu_{GL}LQ}{\pi g_c[(p_i - p_o) + \dfrac{g}{g_c}\rho_{GL}(h_i - h_o)]}}. \qquad \text{(Eq. 5)}$$

The data calculated earlier, Sections D.2-D.4, and L = 24 in (Table A-2, Appendix A) can be used:

$$\mathbf{D^4 = \frac{(128)(38.49)cP(6.72x10^{-4})\dfrac{lb}{ft-s-cP}(24)in(4.47x10^{-7})\dfrac{ft^3(12)^3in^3}{s(1)ft^3}}{\pi(32.17)\dfrac{ft-lb}{lb_f-s^2}(226)\dfrac{lb_f}{ft^2}}}$$

$D^4 = 2.687x10^{-6}$ in^4

D = 0.04049 in.

The value of the “dynamic” capillary diameter for the other manometer readings in Table A-2 (Appendix A) were calculated in the same way as above. Resulting in:

D = 0.0405, 0.0333, 0.0394, 0.0402, 0.0380, 0.0402, and 0.0398 in.

The second value is way out of line with the other six and will be neglected. Thus, (Pintar, 2001):

$$\mathbf{\overline{D} = \frac{(0.0405 + 0.0394 + 0.0402 + 0.0380 + 0.0402 + 0.0398)\ in}{6} = 0.0397\ in}$$

$$\mathbf{s_D = \sqrt{\frac{[(8)^2 + (3)^2 + (5)^2 + (17)^2 + (5)^2 + (1)^2](10^{-8})}{5}} = 0.00091\ in}.$$

D.6. Error Analysis of "Dynamic" Capillary Diameter

The dominant reading error is associated with the poor temperature control of the capillary viscometer. The temperatures in Table A-2 (Appendix A) are probably ±1 °C. This will lead to uncertainty in the viscosity of the glycerol solution in Eq. 5. The reading error associated with viscosity can be estimated from (Dean, 1973, p. 10-289, see also Pintar, 2001):

$$\mathbf{e_{R_{\mu GL}} = \left|\frac{\partial \mu_{GL}}{\partial T}\right| e_T = \left|\frac{31.62 - 55.47}{30 - 20}\right| \frac{cP}{^oC}(1)^o C = 2.385 cP}.$$

The data used in Section D.5 will be used to estimate the reading error in D. By differentiating Eq. 5 (Sec. D.5) with respect to viscosity (μ_{GL}) and rearranging:

$$\mathbf{e_{R_{D^4}} = \left|\frac{\partial D^4}{\partial \mu_{GL}}\right| e_{R_{\mu GL}} = \overline{D^4}\,\frac{e_{R_{\mu GL}}}{\mu_{GL}} = (0.0397)^4 in^4 \frac{(2.385)cP}{(38.49)cP} = 1.539x10^{-7}\ in^4}$$

$$\mathbf{e_{R_D} = \left|\frac{\partial D}{\partial (D^4)}\right| e_{R_{D^4}} = \left|\frac{1}{4}[(D^4)]^{-3/4}\right| e_{R_{D^4}} = \frac{(1.539x10^{-7})in^4}{4(0.0397)^3 in^3} = 0.00061\ in}.$$

Since the reading error is of the same order of magnitude as the standard deviation (Pintar, 2001):

$$\mathbf{e_D = \frac{1}{2}\left(\frac{s_D}{\sqrt{n}} + \frac{e_{R_D}}{\sqrt{3}}\right) = \frac{1}{2}\left(\frac{0.00091}{\sqrt{6}} + \frac{0.00061}{\sqrt{3}}\right) in = 0.00036}.$$

Thus, the "dynamic" capillary diameter is 0.0397±0.00036 in.